CHAPTER 4

Proportional Relationships

Why Learn This?

Proportions can be used to find the heights of objects that are too tall to measure directly, such as a lighthouse.

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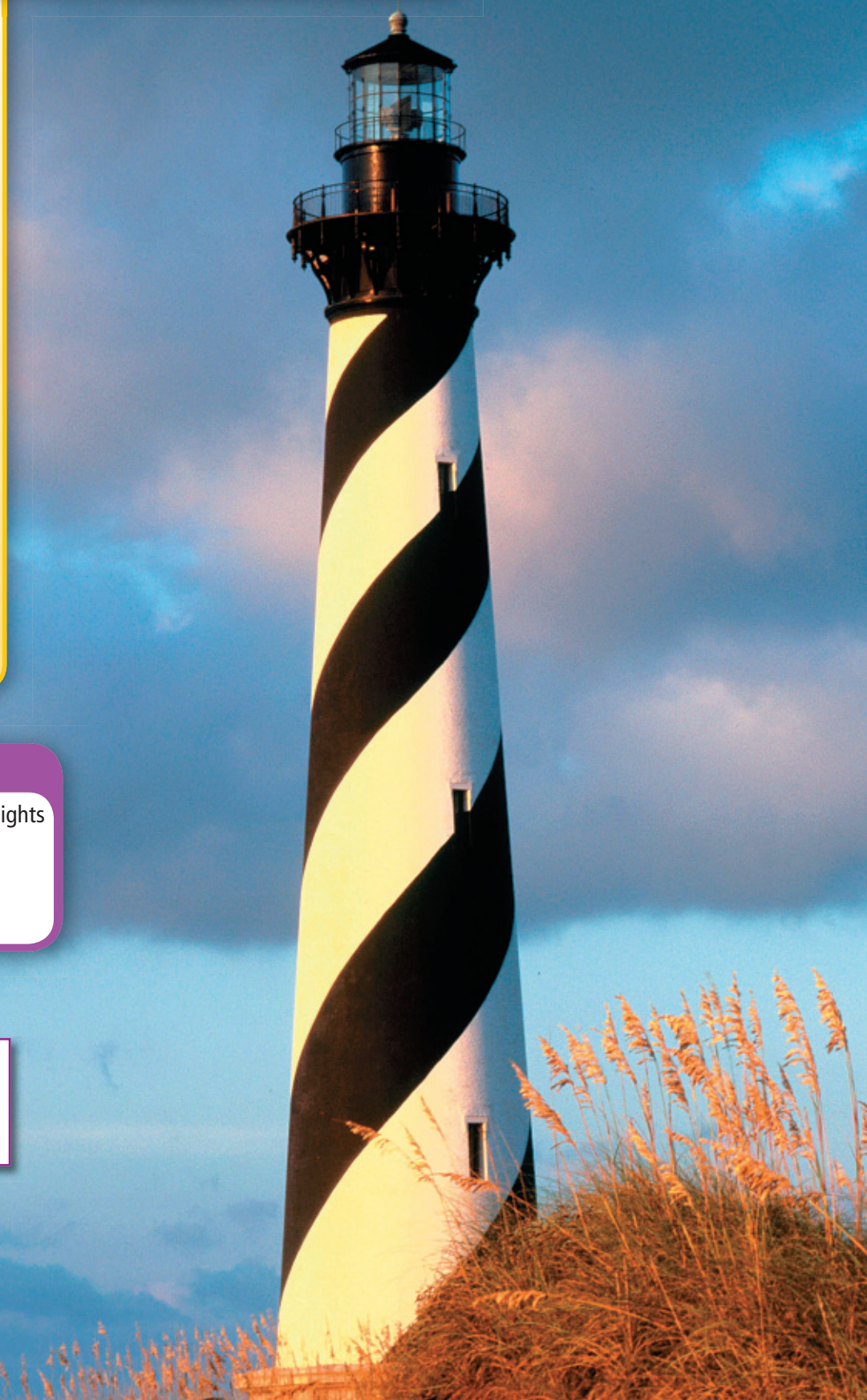
Chapter Focus

- Use proportionality to solve problems, including problems involving similar objects, units of measurement, and rates.



Are You Ready?

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Vocabulary

Choose the best term from the list to complete each sentence.

equation
evaluate
irrational number
rational number
solution

1. A(n) __?__ states that two expressions are equivalent.
2. To __?__ an expression is to substitute a number for the variable and simplify.
3. A value of the variable in an equation that makes the statement true is a(n) __?__ of the equation.
4. A(n) __?__ is a number that can be written as a ratio of two integers.

Complete these exercises to review skills you will need for this chapter.

Evaluate Expressions

Evaluate each expression.

5. $x + 5$ for $x = -18$
6. $-9y$ for $y = 13$
7. $\frac{z}{-6}$ for $z = 96$
8. $w - 9$ for $w = -13$
9. $-3z + 1$ for $z = 4$
10. $3w + 9$ for $w = 7$
11. $5 - \frac{y}{3}$ for $y = -3$
12. $x^2 + 1$ for $x = -2$

Solve Equations

Solve each equation.

13. $y + 14 = -3$
14. $-4y = -72$
15. $y - 6 = 39$
16. $\frac{y}{3} = -9$
17. $56 = 8y$
18. $26 = y + 2$
19. $25 - y = 7$
20. $\frac{121}{y} = 11$
21. $-72 = 3y$
22. $25 = \frac{150}{y}$
23. $15 + y = 4$
24. $-120 = -2y$

Number Patterns

Find the next three numbers in the pattern.

25. 95, 112, 129, 146, . . .
26. 85, 65, 60, 40, 35, . . .
27. 20, 20, 100, 100, 500, . . .
28. 12, 14, 17, 21, 26, . . .
29. 1, 3, 5, 7, . . .
30. −19, −12, −5, 2, . . .
31. 5, −10, 20, −40, 80, . . .
32. 0, −10, −5, −15, −10, . . .

CHAPTER 4

Study Guide: Preview

Where You've Been

Previously, you

- used ratios to describe proportional situations.
- used ratios to make predictions in proportional situations.
- used tables to describe proportional relationships involving conversions.

In This Chapter

You will study

- using division to find unit rates and ratios in proportional relationships.
- estimating and finding solutions to application problems involving proportional relationships.
- generating formulas involving unit conversions.
- using critical attributes to define similarity.
- using ratios and proportions in scale drawings and scale models.

Where You're Going

You can use the skills learned in this chapter

- to read and interpret maps.
- to find heights of objects that are too tall to measure.

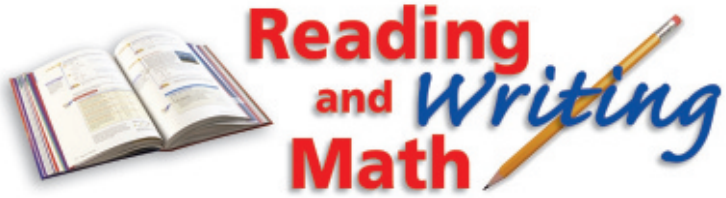
Key Vocabulary/Vocabulario

corresponding angles	ángulos correspondientes
corresponding sides	lados correspondientes
equivalent ratios	razones equivalentes
proportion	proporción
rate	tasa
ratio	razón
scale	escala
scale drawing	dibujo a escala
scale model	modelo a escala
similar	semejante

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. "Miles per hour," "students per class," and "Calories per serving" are all examples of *rates*. What other rates can you think of? How would you describe a **rate** to someone if you couldn't use examples in your explanation?
2. You can select a gear ratio on a bicycle for maximum speed. Think of other examples where the word **ratio** is used. What do these examples have in common?
3. *Similar* means "having characteristics in common." If two triangles are **similar**, what might they have in common?



CHAPTER 4

Writing Strategy: Use Your Own Words

Using your own words to explain a concept can help you understand the concept. For example, learning how to solve equations might seem difficult if the textbook does not explain solving equations in the same way that you would.

As you work through each lesson:

- Identify the important ideas from the explanation in the book.
- Use your own words to explain these ideas.

What Sara Reads

An **equation** is a mathematical statement that two expressions are equal in value.

When an equation contains a variable, a value of the variable that makes the statement true is called a **solution** of the equation.

If a variable is multiplied by a number, you can often use division to isolate the variable. Divide both sides of the equation by the number.

What Sara Writes

An equation has an equal sign to show that two expressions are equal to each other.

The solution of an equation that has a variable in it is the number that the variable is equal to.

When the variable is multiplied by a number, you can undo the multiplication and get the variable alone by dividing both sides of the equation by the number.

Try This

Rewrite each sentence in your own words.

1. When solving addition equations involving integers, isolate the variable by adding opposites.
2. When you solve equations that have one operation, you use an inverse operation to isolate the variable.

4-1 Ratios

Learn to identify, write, and compare ratios.

Vocabulary
ratio

In basketball practice, Kathlene made 17 baskets in 25 attempts. She compared the number of baskets she made to the total number of attempts she made by using the *ratio* $\frac{17}{25}$. A **ratio** is a comparison of two quantities by division.

Kathlene can write her ratio of baskets made to attempts in three different ways.

$\frac{17}{25}$ 17 to 25 17:25

EXAMPLE 1 Writing Ratios

A basket of fruit contains 6 apples, 4 bananas, and 3 oranges. Write each ratio in all three forms.

A bananas to apples

$$\frac{\text{number of bananas}}{\text{number of apples}} = \frac{4}{6}$$

There are 4 bananas and 6 apples.

The ratio of bananas to apples can be written as $\frac{4}{6}$, 4 to 6, or 4:6.

B bananas and apples to oranges

$$\frac{\text{number of bananas and apples}}{\text{number of oranges}} = \frac{4+6}{3} = \frac{10}{3}$$

The ratio of bananas and apples to oranges can be written as $\frac{10}{3}$, 10 to 3, or 10:3.

C oranges to total pieces of fruit

$$\frac{\text{number of oranges}}{\text{number of total pieces of fruit}} = \frac{3}{6+4+3} = \frac{3}{13}$$

The ratio of oranges to total pieces of fruit can be written as $\frac{3}{13}$, 3 to 13, or 3:13.

Sometimes a ratio can be simplified. To simplify a ratio, first write it in fraction form and then simplify the fraction.

EXAMPLE 2 Writing Ratios in Simplest Form

At Franklin Middle School, there are 252 students in the seventh grade and 9 seventh-grade teachers. Write the ratio of students to teachers in simplest form.

$\frac{\text{students}}{\text{teachers}} = \frac{252}{9}$ *Write the ratio as a fraction.*

$= \frac{252 \div 9}{9 \div 9}$ *Simplify.*

$= \frac{28}{1}$ *For every 28 students, there is 1 teacher.*

The ratio of students to teachers is 28 to 1.

Remember!

A fraction is in simplest form when the GCF of the numerator and denominator is 1.

To compare ratios, write them as fractions with common denominators. Then compare the numerators.

EXAMPLE 3 Comparing Ratios

Tell whether the wallet size photo or the portrait size photo has the greater ratio of width to length.

	Width (in.)	Length (in.)
Wallet	3.5	5
Personal	4	6
Desk	5	7
Portrait	8	10

Wallet: $\frac{\text{width (in.)}}{\text{length (in.)}} = \frac{3.5}{5}$

Portrait: $\frac{\text{width (in.)}}{\text{length (in.)}} = \frac{8}{10} = \frac{4}{5}$

Write the ratios as fractions with common denominators.

Because $4 > 3.5$ and the denominators are the same, the portrait size photo has the greater ratio of width to length.

Think and Discuss

1. **Explain** why the ratio $\frac{10}{3}$ in Example 1B is not written as a mixed number.
2. **Tell** how to simplify a ratio.
3. **Explain** how to compare two ratios.

4-1 Exercises

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Exercises 1–10, 11, 15, 17, 19

GUIDED PRACTICE

See Example 1

Sun-Li has 10 blue marbles, 3 red marbles, and 17 white marbles. Write each ratio in all three forms.

1. blue marbles to red marbles
2. red marbles to total marbles

See Example 2

3. In a 40-gallon aquarium, there are 21 neon tetras and 7 zebra danio fish. Write the ratio of neon tetras to zebra danio fish in simplest form.

See Example 3

4. Tell whose DVD collection has the greater ratio of comedy movies to adventure movies.

	Joseph	Yolanda
Comedy	5	7
Adventure	3	5

INDEPENDENT PRACTICE

See Example 1

A soccer league has 25 sixth-graders, 30 seventh-graders, and 15 eighth-graders. Write each ratio in all three forms.

5. 6th-graders to 7th-graders
6. 6th-graders to total students
7. 7th-graders to 8th-graders
8. 7th- and 8th-graders to 6th-graders

See Example 2

9. Thirty-six people auditioned for a play, and 9 people got roles. Write the ratio in simplest form of the number of people who auditioned to the number of people who got roles.

See Example 3

10. Tell whose bag of nut mix has the greater ratio of peanuts to total nuts.

	Dina	Don
Almonds	6	11
Cashews	8	7
Peanuts	10	18

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP11.

Use the table for Exercises 11–13.

11. Tell whether group 1 or group 2 has the greater ratio of the number of people for an open-campus lunch to the number of people with no opinion.

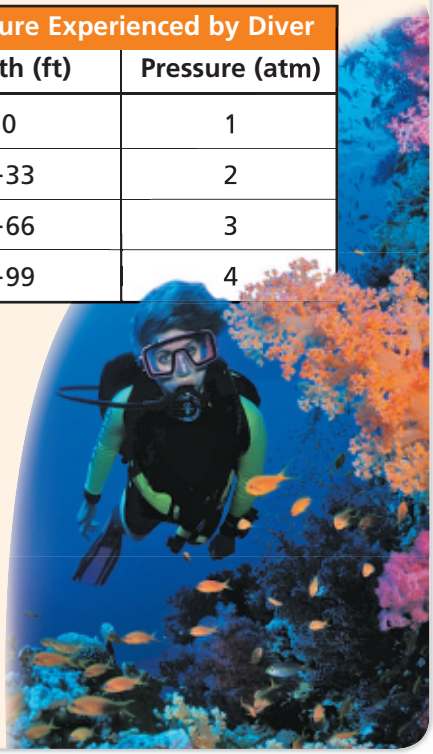
Opinions on Open-Campus Lunch			
	Group 1	Group 2	Group 3
For	9	10	12
Against	14	16	16
No Opinion	5	6	8

12. Which group has the least ratio of the number of people against an open-campus lunch to the total number of survey responses?
13. **Estimation** For each group, is the ratio of the number of people for an open-campus lunch to the number of people against it less than or greater than $\frac{1}{2}$?

Physical Science LINK

The pressure of water at different depths can be measured in *atmospheres*, or atm. The water pressure on a scuba diver increases as the diver descends below the surface. Use the table for Exercises 14–20.

Pressure Experienced by Diver	
Depth (ft)	Pressure (atm)
0	1
−33	2
−66	3
−99	4



Write each ratio in all three forms.

14. pressure at −33 ft to pressure at surface

15. pressure at −66 ft to pressure at surface

16. pressure at −99 ft to pressure at surface

17. pressure at −66 ft to pressure at −33 ft

18. pressure at −99 ft to pressure at −66 ft

19. Tell whether the ratio of pressure at −66 ft to pressure at −33 ft is greater than or less than the ratio of pressure at −99 ft to pressure at −66 ft.

20. **Challenge** Compare the ratio of the beginning pressure and the new pressure when a scuba diver goes from −33 ft to −66 ft and when the diver goes from the surface to −33 ft. Are these ratios of pressures less than or greater than the ratio of pressure when the diver goes from −66 ft to −99 ft? Use ratios to explain.

Test Prep and Spiral Review

21. **Multiple Choice** Johnson Middle School has 125 sixth-graders, 150 seventh-graders, and 100 eighth-graders. Which statement is NOT true?

(A) The ratio of sixth-graders to seventh-graders is 5 to 6.

(B) The ratio of eighth-graders to seventh-graders is 3:2.

(C) The ratio of sixth-graders to students in all three grades is 1:3.

(D) The ratio of eighth-graders to students in all three grades is 4 to 15.

22. **Short Response** A pancake recipe calls for 4 cups of pancake mix for every 3 cups of milk. A biscuit recipe calls for 2 cups of biscuit mix for every 1 cup of milk. Which recipe has a greater ratio of mix to milk? Explain.

Solve. (Lesson 3-5)

23. $1.23 + x = -5.47$ **24.** $3.8y = 27.36$ **25.** $v - 3.8 = 4.7$

26. On Monday Jessika ran $3\frac{1}{2}$ miles. On Wednesday she ran $4\frac{1}{3}$ miles. How much farther did Jessika run on Wednesday? (Lesson 3-7)

4-2 Rates

Learn to find and compare unit rates, such as average speed and unit price.

Vocabulary

rate

unit rate

The Lawsons are going camping at Rainbow Falls, which is 288 miles from their home. They would like to reach the campground in 6 hours. What should their average speed be in miles per hour?

In order to answer the question above, you need to find the family's *rate* of travel. A **rate** is a ratio that compares two quantities measured in different units.

The Lawson family's rate is $\frac{288 \text{ miles}}{6 \text{ hours}}$.

A **unit rate** is a rate whose denominator is 1 when it is written as a fraction. To change a rate to a unit rate, first write the rate as a fraction and then divide both the numerator and denominator by the denominator.

EXAMPLE 1 Finding Unit Rates

Interactivities Online ▶

A **During exercise, Sonia's heart beats 675 times in 5 minutes. How many times does it beat per minute?**

$\frac{675 \text{ beats}}{5 \text{ minutes}}$ *Write a rate that compares heart beats and time.*

$\frac{675 \text{ beats} \div 5}{5 \text{ minutes} \div 5}$ *Divide the numerator and denominator by 5.*

$\frac{135 \text{ beats}}{1 \text{ minute}}$ *Simplify.*

Sonia's heart beats 135 times per minute.

B **To make 4 large pizza pockets, Paul needs 14 cups of broccoli. How much broccoli does he need for 1 large pizza pocket?**

$\frac{14 \text{ cups broccoli}}{4 \text{ pizza pockets}}$ *Write a rate that compares cups to pockets.*

$\frac{14 \text{ cups broccoli} \div 4}{4 \text{ pizza pockets} \div 4}$ *Divide the numerator and denominator by 4.*

$\frac{3.5 \text{ cups broccoli}}{1 \text{ pizza pocket}}$ *Simplify.*

Paul needs 3.5 cups of broccoli to make 1 large pizza pocket.

An average rate of speed is the ratio of distance traveled to time. The ratio is a rate because the units being compared are different.

EXAMPLE 2 Finding Average Speed

The Lawsons want to drive 288 miles to Rainbow Falls in 6 hours. What should their average speed be in miles per hour?

$\frac{288 \text{ miles}}{6 \text{ hours}}$ *Write the rate as a fraction.*

$\frac{288 \text{ miles} \div 6}{6 \text{ hours} \div 6} = \frac{48 \text{ miles}}{1 \text{ hour}}$ *Divide the numerator and denominator by the denominator.*

Their average speed should be 48 miles per hour.

A unit price is the price of one unit of an item. The unit used depends on how the item is sold. The table shows some examples.

Type of Item	Examples of Units
Liquid	Fluid ounces, quarts, gallons, liters
Solid	Ounces, pounds, grams, kilograms
Any item	Bottle, container, carton

EXAMPLE 3 *Consumer Math Application*

The Lawsons stop at a roadside farmers' market. The market offers lemonade in three sizes. Which size lemonade has the lowest price per fluid ounce?

Size	Price
12 fl oz	$0.89
18 fl oz	$1.69
24 fl oz	$2.09

Divide the price by the number of fluid ounces (fl oz) to find the unit price of each size.

$\frac{\$0.89}{12 \text{ fl oz}} \approx \frac{\$0.07}{\text{fl oz}}$ $\frac{\$1.69}{18 \text{ fl oz}} \approx \frac{\$0.09}{\text{fl oz}}$ $\frac{\$2.09}{24 \text{ fl oz}} \approx \frac{\$0.09}{\text{fl oz}}$

Since $\$0.07 < \$0.09$, the 12 fl oz lemonade has the lowest price per fluid ounce.

Think and Discuss

1. **Explain** how you can tell whether a rate represents a unit rate.
2. **Suppose** a store offers cereal with a unit price of $0.15 per ounce. Another store offers cereal with a unit price of $0.18 per ounce. Before determining which is the better buy, what variables must you consider?

4-2 Exercises

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Exercises 1–8, 9, 11, 13, 23

GUIDED PRACTICE

See Example 1

1. A faucet leaks 668 milliliters of water in 8 minutes. How many milliliters of water does the faucet leak per minute?

2. A recipe for 6 muffins calls for 360 grams of oat flakes. How many grams of oat flakes are needed for each muffin?

See Example 2

3. An airliner makes a 2,748-mile flight in 6 hours. What is the airliner's average rate of speed in miles per hour?

See Example 3

4. **Consumer Math** During a car trip, the Webers buy gasoline at three different stations. At the first station, they pay $18.63 for 9 gallons of gas. At the second, they pay $29.54 for 14 gallons. At the third, they pay $33.44 for 16 gallons. Which station offers the lowest price per gallon?

INDEPENDENT PRACTICE

See Example 1

5. An after-school job pays $116.25 for 15 hours of work. How much money does the job pay per hour?

6. It took Samantha 324 minutes to cook an 18 lb turkey. How many minutes per pound did it take to cook the turkey?

See Example 2

7. **Sports** The first Indianapolis 500 auto race took place in 1911. The winning car covered the 500 miles in 6.7 hours. What was the winning car's average rate of speed in miles per hour?

See Example 3

8. **Consumer Math** A supermarket sells orange juice in three sizes. The 32 fl oz container costs $1.99, the 64 fl oz container costs $3.69, and the 96 fl oz container costs $5.85. Which size orange juice has the lowest price per fluid ounce?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP11.

Find each unit rate. Round to the nearest hundredth, if necessary.

9. 9 runs in 3 games
10. $207,000 for 1,800 ft^2
11. $2,010 in 6 mo
12. 52 songs on 4 CDs
13. 226 mi on 12 gal
14. 324 words in 6 min
15. 12 hr for $69
16. 6 lb for $12.96
17. 488 mi in 4 trips
18. 220 m in 20 s
19. 1.5 mi in 39 min
20. 24,000 km in 1.5 hr

21. In Grant Middle School, each class has an equal number of students. There are 38 classes and a total of 1,026 students. Write a rate that describes the distribution of students in the classes at Grant. What is the unit rate?

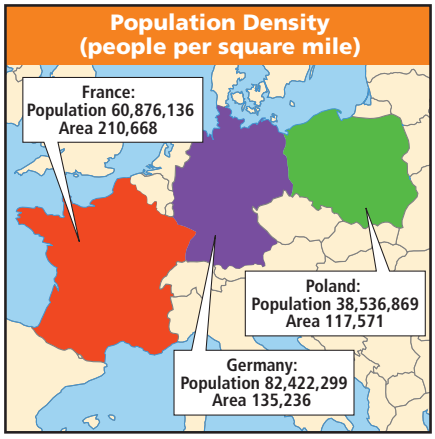
22. **Estimation** Use estimation to determine which is the better buy: 450 minutes of phone time for $49.99 or 800 minutes for $62.99.

Find each unit price. Then decide which is the better buy.

23. $\frac{\$2.52}{42\text{ oz}}$ or $\frac{\$3.64}{52\text{ oz}}$ **24.** $\frac{\$28.40}{8\text{ yd}}$ or $\frac{\$55.50}{15\text{ yd}}$ **25.** $\frac{\$8.28}{0.3\text{ m}}$ or $\frac{\$13.00}{0.4\text{ m}}$

26. Sports At the track meet, Justin won the 100-meter race in 12.61 seconds. Shawn won the 200-meter race in 26.38 seconds. Which runner ran at a faster average rate?

27. Social Studies The population density of a country is the average number of people per unit of area. Write the population densities of the countries in the map at right as unit rates. Round your answers to the nearest person per square mile. Then rank the countries from least population density to greatest population density.



28. Write a Problem A store sells paper towels in packs of 6 and packs of 8. Use this information to write a problem about comparing unit rates.

29. Write About It Michael Jordan has the highest scoring average in NBA history. During his career, he played in 1,072 games and scored a total of 32,292 points. Explain how to find a unit rate to describe his scoring average. What is the unit rate?

30. Challenge Mike fills his car's gas tank with 20 gallons of regular gas at $2.01 per gallon. His car averages 25 miles per gallon. Serena fills her car's tank with 15 gallons of premium gas at $2.29 per gallon. Her car averages 30 miles per gallon. Compare the drivers' unit costs of driving one mile.

Test Prep and Spiral Review

31. Multiple Choice What is the unit price of a 16-ounce box of cereal that sells for $2.48?

Ⓐ $0.14 Ⓑ $0.15 Ⓒ $0.0155 Ⓓ $0.155

32. Short Response A carpenter needs 3 minutes to make 5 cuts in a board. Each cut takes the same length of time. At what rate is the carpenter cutting?

Multiply. Estimate to check whether each answer is reasonable. (Lesson 3-3)

33. $-4.87 \cdot (-2.4)$ **34.** $-6.2 \cdot 130$ **35.** $0.65 \cdot (-2.07)$

36. Julita's walking stick is $3\frac{2}{3}$ feet long, and Toni's walking stick is $3\frac{3}{8}$ feet long. Whose walking stick is longer and by how much? (Lesson 3-8)

4-3 Identifying and Writing Proportions

Learn to find equivalent ratios and to identify proportions.

Vocabulary

equivalent ratios

proportion

Students in Mr. Howell's math class are measuring the width w and the length ℓ of their faces. The ratio of ℓ to w is 6 inches to 4 inches for Jean and 21 centimeters to 14 centimeters for Pat.

Oval face: $\frac{\ell}{w} \approx \frac{3}{2}$

Round face: $\ell \approx w$

These ratios can be written as $\frac{6}{4}$ and $\frac{21}{14}$. Since both ratios simplify to $\frac{3}{2}$, they are equivalent. **Equivalent ratios** are ratios that name the same comparison.

An equation stating that two ratios are equivalent is called a **proportion**. The equation, or proportion, below states that the ratios $\frac{6}{4}$ and $\frac{21}{14}$ are equivalent.

$$\frac{6}{4} = \frac{21}{14}$$

If two ratios are equivalent, they are said to be *proportional*, or *in proportion*.

Reading Math

Read the proportion $\frac{6}{4} = \frac{21}{14}$ by saying "six is to four as twenty-one is to fourteen."

EXAMPLE 1 Comparing Ratios in Simplest Form

Determine whether the ratios are proportional.

A $\frac{2}{7}, \frac{6}{21}$

$\frac{2}{7}$ — $\frac{2}{7}$ *is already in simplest form.*

$\frac{6}{21} = \frac{6 \div 3}{21 \div 3} = \frac{2}{7}$ — *Simplify* $\frac{6}{21}$.

Since $\frac{2}{7} = \frac{2}{7}$, the ratios are proportional.

B $\frac{8}{24}, \frac{6}{20}$

$\frac{8}{24} = \frac{8 \div 8}{24 \div 8} = \frac{1}{3}$ — *Simplify* $\frac{8}{24}$.

$\frac{6}{20} = \frac{6 \div 2}{20 \div 2} = \frac{3}{10}$ — *Simplify* $\frac{6}{20}$.

Since $\frac{1}{3} \neq \frac{3}{10}$, the ratios are *not* proportional.

EXAMPLE 2 Comparing Ratios Using a Common Denominator

Use the data in the table to determine whether the ratios of oats to water are proportional for both servings of oatmeal.

Servings of Oatmeal	Cups of Oats	Cups of Water
8	2	4
12	3	6

Write the ratios of oats to water for 8 servings and for 12 servings.

Ratio of oats to water, 8 servings: $\frac{2}{4}$ *Write the ratio as a fraction.*

Ratio of oats to water, 12 servings: $\frac{3}{6}$ *Write the ratio as a fraction.*

$\frac{2}{4} = \frac{2 \cdot 6}{4 \cdot 6} = \frac{12}{24}$

$\frac{3}{6} = \frac{3 \cdot 4}{6 \cdot 4} = \frac{12}{24}$

Write the fractions with a common denominator, such as 24.

Since both ratios are equal to $\frac{12}{24}$, they are proportional.

You can find an equivalent ratio by multiplying or dividing both terms of a ratio by the same number.

EXAMPLE 3 Finding Equivalent Ratios and Writing Proportions

Find a ratio equivalent to each ratio. Then use the ratios to write a proportion.

A $\frac{8}{14}$

$\frac{8}{14} = \frac{8 \cdot 20}{14 \cdot 20} = \frac{160}{280}$ *Multiply both terms by any number, such as 20.*

$\frac{8}{14} = \frac{160}{280}$ *Write a proportion.*

B $\frac{4}{18}$

$\frac{4}{18} = \frac{4 \div 2}{18 \div 2} = \frac{2}{9}$ *Divide both terms by a common factor, such as 2.*

$\frac{4}{18} = \frac{2}{9}$ *Write a proportion.*

Life Science LINK

The ratios of the sizes of the segments of a nautilus shell are approximately equal to the *golden ratio*, 1.618. . . . This ratio can be found in many places in nature.

Think and Discuss

1. **Explain** why the ratios in Example 1B are not proportional.
2. **Describe** what it means for ratios to be proportional.
3. **Give an example** of a proportion. Then tell how you know it is a proportion.

4-3 Exercises

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Exercises 1–28, 29, 39, 41, 43

GUIDED PRACTICE

See Example 1 **Determine whether the ratios are proportional.**

1. $\frac{2}{3}, \frac{4}{6}$
2. $\frac{5}{10}, \frac{8}{18}$
3. $\frac{9}{12}, \frac{15}{20}$
4. $\frac{3}{4}, \frac{8}{12}$

See Example 2

5. $\frac{10}{12}, \frac{15}{18}$
6. $\frac{6}{9}, \frac{8}{12}$
7. $\frac{3}{4}, \frac{5}{6}$
8. $\frac{4}{6}, \frac{6}{9}$

See Example 3 **Find a ratio equivalent to each ratio. Then use the ratios to write a proportion.**

9. $\frac{1}{3}$
10. $\frac{9}{21}$
11. $\frac{8}{3}$
12. $\frac{10}{4}$

INDEPENDENT PRACTICE

See Example 1 **Determine whether the ratios are proportional.**

13. $\frac{5}{8}, \frac{7}{14}$
14. $\frac{8}{24}, \frac{10}{30}$
15. $\frac{18}{20}, \frac{81}{180}$
16. $\frac{15}{20}, \frac{27}{35}$

See Example 2

17. $\frac{2}{3}, \frac{4}{9}$
18. $\frac{18}{12}, \frac{15}{10}$
19. $\frac{7}{8}, \frac{14}{24}$
20. $\frac{18}{54}, \frac{10}{30}$

See Example 3 **Find a ratio equivalent to each ratio. Then use the ratios to write a proportion.**

21. $\frac{5}{9}$
22. $\frac{27}{60}$
23. $\frac{6}{15}$
24. $\frac{121}{99}$
25. $\frac{11}{13}$
26. $\frac{5}{22}$
27. $\frac{78}{104}$
28. $\frac{27}{72}$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP11.

Complete each table of equivalent ratios.

29.

angelfish	4	8	■	20
tiger fish	■	6	18	■

30.

squares	2	4	6	8
circles	■	16	■	■

Find two ratios equivalent to each given ratio.

31. 3 to 7
32. 6:2
33. $\frac{5}{12}$
34. 8:4
35. 6 to 9
36. $\frac{10}{50}$
37. 10:4
38. 1 to 10

39. **Ecology** If you recycle one aluminum can, you save enough energy to run a TV for four hours.
    a. Write the ratio of cans to hours.
    b. Marti's class recycled enough aluminum cans to run a TV for 2,080 hours. Did the class recycle 545 cans? Justify your answer using equivalent ratios.

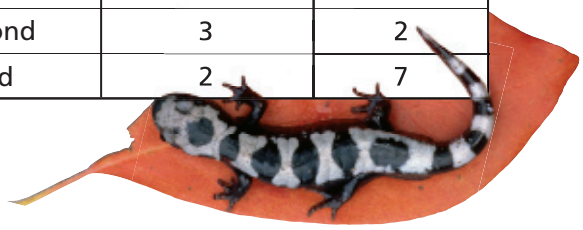
40. **Critical Thinking** The ratio of girls to boys riding a bus is 15:12. If the driver drops off the same number of girls as boys at the next stop, does the ratio of girls to boys remain 15:12? Explain.

41. Critical Thinking Write all possible proportions using only the numbers 1, 2, and 4.

42. School Last year in Kerry's school, the ratio of students to teachers was 22:1. Write an equivalent ratio to show how many students and teachers there could have been at Kerry's school.

43. Life Science Students in a biology class visited four different ponds to determine whether salamanders and frogs were inhabiting the area.

Pond	Number of Salamanders	Number of Frogs
Cypress Pond	8	5
Mill Pond	15	10
Clear Pond	3	2
Gill Pond	2	7



a. What was the ratio of salamanders to frogs in Cypress Pond?

b. In which two ponds was the ratio of salamanders to frogs the same?

44. Marcus earned $230 for 40 hours of work. Phillip earned $192 for 32 hours of work. Are these pay rates proportional? Explain.

45. What's the Error? A student wrote the proportion $\frac{13}{20} = \frac{26}{60}$. What did the student do wrong?

46. Write About It Explain two different ways to determine if two ratios are proportional.

47. Challenge A skydiver jumps out of an airplane. After 0.8 second, she has fallen 100 feet. After 3.1 seconds, she has fallen 500 feet. Is the rate (in feet per second) at which she falls the first 100 feet proportional to the rate at which she falls the next 400 feet? Explain.

Test Prep and Spiral Review

48. Multiple Choice Which ratio is NOT equivalent to $\frac{32}{48}$?

Ⓐ $\frac{2}{3}$ Ⓑ $\frac{8}{12}$ Ⓒ $\frac{64}{96}$ Ⓓ $\frac{128}{144}$

49. Multiple Choice Which ratio can form a proportion with $\frac{5}{6}$?

Ⓕ $\frac{13}{18}$ Ⓖ $\frac{25}{36}$ Ⓗ $\frac{70}{84}$ Ⓙ $\frac{95}{102}$

Divide. Estimate to check whether each answer is reasonable. (Lesson 3-4)

50. 14.35 ÷ 0.7 **51.** −9 ÷ 2.4 **52.** 12.505 ÷ 3.05 **53.** 427 ÷ (−5.6)

Compare. Write <, >, or =. (Lesson 4-1)

54. 3:5 ▢ 12:15 **55.** 33:66 ▢ 1:3 **56.** 9:24 ▢ 3:8 **57.** 15:7 ▢ 8:3

4-4 Solving Proportions

Learn to solve proportions by using cross products.

Vocabulary

cross product

Interactivities Online ▶

Density is a ratio that compares a substance's mass to its volume. If you are given the density of ice, you can find the mass of 3 mL of ice by solving a proportion.

For two ratios, the product of the first term in one ratio and the second term in the other is a **cross product**. If the cross products are equal, then the ratios form a proportion.

$\frac{2}{5} = \frac{6}{15}$

$5 \cdot 6 = 30$

$2 \cdot 15 = 30$

Ice floats in water because the density of ice is less than the density of water.

CROSS PRODUCTS

In the proportion $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, the cross products, $a \cdot d$ and $b \cdot c$, are equal.

You can use cross products to solve proportions with variables.

EXAMPLE 1 Solving Proportions Using Cross Products

Use cross products to solve the proportion $\frac{p}{6} = \frac{10}{3}$.

$\frac{p}{6} = \frac{10}{3}$

$10 \cdot 6 = p \cdot 3$ *The cross products are equal.*

$60 = 3p$ *Multiply.*

$\frac{60}{3} = \frac{3p}{3}$ *Divide each side by 3.*

$20 = p$

It is important to set up proportions correctly. Each ratio must compare corresponding quantities in the same order. Suppose a boat travels 16 miles in 4 hours and 8 miles in x hours at the same speed. Either of these proportions could represent this situation.

Trip 1 → $\frac{16 \text{ mi}}{4 \text{ h}} = \frac{8 \text{ mi}}{x \text{ h}}$ ← Trip 2

$\frac{16 \text{ mi}}{8 \text{ mi}} = \frac{4 \text{ h}}{x \text{ h}}$ ← Trip 1, ← Trip 2

EXAMPLE 2 PROBLEM SOLVING APPLICATION

PROBLEM SOLVING

Density is the ratio of a substance's mass to its volume. The density of ice is 0.92 g/mL. What is the mass of 3 mL of ice?

1 Understand the Problem

Rewrite the question as a statement.

- Find the mass, in grams, of 3 mL of ice.

List the **important information:**

- density $= \frac{\text{mass (g)}}{\text{volume (mL)}}$
- density of ice $= \frac{0.92\text{ g}}{1\text{ mL}}$

2 Make a Plan

Set up a proportion using the given information. Let m represent the mass of 3 mL of ice.

$$\frac{0.92\text{ g}}{1\text{ mL}} = \frac{m}{3\text{ mL}} \quad \begin{matrix}\leftarrow \textit{mass} \\ \leftarrow \textit{volume}\end{matrix}$$

3 Solve

Solve the proportion.

$\frac{0.92}{1} = \frac{m}{3}$ *Write the proportion.*

$m \cdot 1 = 0.92 \cdot 3$ *The cross products are equal.*

$m = 2.76$ *Multiply.*

The mass of 3 mL of ice is 2.76 g.

4 Look Back

Since the density of ice is 0.92 g/mL, each milliliter of ice has a mass of a little less than 1 g. So 3 mL of ice should have a mass of a little less than 3 g. Since 2.76 is a little less than 3, the answer is reasonable.

Think and Discuss

1. **Explain** how the term *cross product* can help you remember how to solve a proportion.
2. **Describe** the error in these steps: $\frac{2}{3} = \frac{x}{12}$; $2x = 36$; $x = 18$.
3. **Show** how to use cross products to decide whether the ratios 6:45 and 2:15 are proportional.

4-4 Exercises

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Exercises 1–15, 29, 31, 33, 35, 37, 39

GUIDED PRACTICE

See Example 1

Use cross products to solve each proportion.

1. $\frac{6}{10} = \frac{36}{x}$
2. $\frac{4}{7} = \frac{5}{p}$
3. $\frac{12.3}{m} = \frac{75}{100}$
4. $\frac{t}{42} = \frac{1.5}{3}$

See Example 2

5. A stack of 2,450 one-dollar bills weighs 5 pounds. How much does a stack of 1,470 one-dollar bills weigh?

INDEPENDENT PRACTICE

See Example 1

Use cross products to solve each proportion.

6. $\frac{4}{36} = \frac{x}{180}$
7. $\frac{7}{84} = \frac{12}{h}$
8. $\frac{3}{24} = \frac{r}{52}$
9. $\frac{5}{140} = \frac{12}{v}$
10. $\frac{45}{x} = \frac{15}{3}$
11. $\frac{t}{6} = \frac{96}{16}$
12. $\frac{2}{5} = \frac{s}{12}$
13. $\frac{14}{n} = \frac{5}{8}$

See Example 2

14. Euro coins come in eight denominations. One denomination is the one-euro coin, which is worth 100 cents. A stack of 10 one-euro coins is 21.25 millimeters tall. How tall would a stack of 45 one-euro coins be? Round your answer to the nearest hundredth of a millimeter.

15. There are 18.5 ounces of soup in a can. This is equivalent to 524 grams. Jenna has 8 ounces of soup. How many grams does she have? Round your answer to the nearest whole gram.

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP11.

Solve each proportion. Then find another equivalent ratio.

16. $\frac{4}{h} = \frac{12}{24}$
17. $\frac{x}{15} = \frac{12}{90}$
18. $\frac{39}{4} = \frac{t}{12}$
19. $\frac{5.5}{6} = \frac{16.5}{w}$
20. $\frac{1}{3} = \frac{y}{25.5}$
21. $\frac{18}{x} = \frac{1}{5}$
22. $\frac{m}{4} = \frac{175}{20}$
23. $\frac{8.7}{2} = \frac{q}{4}$
24. $\frac{r}{84} = \frac{32.5}{182}$
25. $\frac{76}{304} = \frac{81}{k}$
26. $\frac{9}{500} = \frac{p}{2{,}500}$
27. $\frac{5}{j} = \frac{6}{19.8}$

28. A certain shade of paint is made by mixing 5 parts blue paint with 2 parts white paint. To get the correct shade, how many quarts of white paint should be mixed with 8.5 quarts of blue paint?

29. **Measurement** If you put an object that has a mass of 40 grams on one side of a balance scale, you would have to put about 18 U.S. dimes on the other side to balance the weight. About how many dimes would balance the weight of a 50-gram object?

30. Sandra drove 126.2 miles in 2 hours at a constant speed. Use a proportion to find how long it would take her to drive 189.3 miles at the same speed.

31. **Multi-Step** In June, a camp has 325 campers and 26 counselors. In July, 265 campers leave and 215 new campers arrive. How many counselors does the camp need in July to keep an equivalent ratio of campers to counselors?

Arrange each set of numbers to form a proportion.

32. 10, 6, 30, 18

33. 4, 6, 10, 15

34. 12, 21, 7, 4

35. 75, 4, 3, 100

36. 30, 42, 5, 7

37. 5, 90, 108, 6

38. Life Science On Monday a marine biologist took a random sample of 50 fish from a pond and tagged them. On Tuesday she took a new sample of 100 fish. Among them were 4 fish that had been tagged on Monday.

a. What comparison does the ratio $\frac{4}{100}$ represent?

b. What ratio represents the number of fish tagged on Monday to n, the total number of fish in the pond?

c. Use a proportion to estimate the number of fish in the pond.

Life Science

This catfish was 7 feet, 7 inches long and weighed 212 pounds! She was caught and re-released in the River Ebro, near Barcelona, Spain.

39. Chemistry The table shows the type and number of atoms in one molecule of citric acid. Use a proportion to find the number of oxygen atoms in 15 molecules of citric acid.

Composition of Citric Acid	
Type of Atom	**Number of Atoms**
Carbon	6
Hydrogen	8
Oxygen	7

40. Earth Science You can find your distance from a thunderstorm by counting the number of seconds between a lightning flash and the thunder. For example, if the time difference is 21 s, then the storm is about 7 km away. About how far away is a storm if the time difference is 9 s?

41. What's the Question? There are 20 grams of protein in 3 ounces of sautéed fish. If the answer is 9 ounces, what is the question?

42. Write About It Give an example from your own life that can be described using a ratio. Then tell how a proportion can give you additional information.

43. Challenge Use the Multiplication Property of Equality and the proportion $\frac{a}{b} = \frac{c}{d}$ to show that the cross product rule works for all proportions.

Test Prep and Spiral Review

44. Multiple Choice Which proportion is correct?

Ⓐ $\frac{4}{8} = \frac{6}{10}$ Ⓑ $\frac{2}{7} = \frac{10}{15}$ Ⓒ $\frac{7}{14} = \frac{15}{30}$ Ⓓ $\frac{16}{25} = \frac{13}{18}$

45. Gridded Response Find a ratio to complete the proportion $\frac{2}{3} = \frac{?}{?}$ so that the cross products are equal to 12. Grid your answer in the form of a fraction.

Estimate. (Lesson 3-1)

46. $16.21 - 14.87$

47. $3.82 \cdot (-4.97)$

48. $-8.7 \cdot (-20.1)$

Find each unit rate. (Lesson 4-2)

49. 128 miles in 2 hours

50. 9 books in 6 weeks

51. $114 in 12 hours

CHAPTER 4 SECTION 4A

Ready To Go On?

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Quiz for Lessons 4-1 Through 4-4

4-1 Ratios

1. The 2007 record for the University of North Carolina softball team was 46 wins to 21 losses. Write the ratio of wins to losses in all three forms.
2. A concession stand sold 14 strawberry, 18 banana, 8 grape, and 6 orange fruit drinks during a game. Tell whether the ratio of strawberry to orange drinks or the ratio of banana to grape drinks is greater.

4-2 Rates

Find each unit rate. Round to the nearest hundredth, if necessary.

3. \$140 for 18 ft^2
4. 346 mi on 22 gal
5. 14 lb for \$2.99
6. Shaunti drove 621 miles in 11.5 hours. What was her average speed in miles per hour?
7. A grocery store sells a 7 oz bag of raisins for \$1.10 and a 9 oz bag of raisins for \$1.46. Which size bag has the lower price per ounce?

4-3 Identifying and Writing Proportions

Find a ratio equivalent to each ratio. Then use the ratios to write a proportion.

8. $\frac{10}{16}$
9. $\frac{21}{28}$
10. $\frac{12}{25}$
11. $\frac{40}{48}$
12. Ryan earned \$272 for 40 hours of work. Jonathan earned \$224 for 32 hours of work. Are these pay rates proportional? Explain.
13. On a given day, the ratio of dollars to euros was approximately 1:0.735. Is the ratio 20 to 14.70 an equivalent ratio? Explain.

4-4 Solving Proportions

Use cross products to solve each proportion.

14. $\frac{n}{8} = \frac{15}{4}$
15. $\frac{20}{t} = \frac{2.5}{6}$
16. $\frac{6}{11} = \frac{0.12}{z}$
17. $\frac{15}{24} = \frac{x}{10}$
18. One human year is said to be about 7 dog years. Cliff's dog is 5.5 years old in human years. Estimate his dog's age in dog years.

Focus on Problem Solving

Make a Plan

- **Choose a problem-solving strategy**

The following are strategies that you might choose to help you solve a problem:

- Make a table
- Find a pattern
- Make an organized list
- Work backward
- Use a Venn diagram
- Draw a diagram
- Guess and test
- Use logical reasoning
- Solve a simpler problem
- Make a model

Tell which strategy from the list above you would use to solve each problem. Explain your choice.

1. A recipe for blueberry muffins calls for 1 cup of milk and 1.5 cups of blueberries. Ashley wants to make more muffins than the recipe yields. In Ashley's muffin batter, there are 4.5 cups of blueberries. If she is using the recipe as a guide, how many cups of milk will she need?

2. There are 32 students in Samantha's math class. Of those students 18 are boys. Write the ratio in simplest form of the number of girls in Samantha's class to the number of boys.

3. Jeremy is the oldest of four brothers. Each of the four boys gets an allowance for doing chores at home each week. The amount of money each boy receives depends on his age. Jeremy is 13 years old, and he gets $12.75. His 11-year-old brother gets $11.25, and his 9-year-old brother gets $9.75. How much money does his 7-year-old brother get?

4. According to an article in a medical journal, a healthful diet should include a ratio of 2.5 servings of meat to 4 servings of vegetables. If you eat 7 servings of meat per week, how many servings of vegetables should you eat?

4-5 Customary Measurements

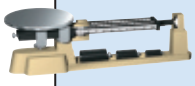
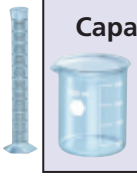
Learn to identify and convert customary units of measure.

The king cobra is one of the world's most poisonous snakes. Just 2 fluid ounces of the snake's venom is enough to kill a 2-ton elephant.

You can use the following benchmarks to help you understand fluid ounces, tons, and other customary units of measure.

Helpful Hint

For more on measurements, see the table of measures on the inside back cover.

	Customary Unit	Benchmark
Length 	Inch (in.)	Length of a small paper clip
	Foot (ft)	Length of a standard sheet of paper
	Mile (mi)	Length of 4 laps around a track
Weight 	Ounce (oz)	Weight of a slice of bread
	Pound (lb)	Weight of 3 apples
	Ton	Weight of a buffalo
Capacity 	Fluid ounce (fl oz)	Amount of water in 2 tablespoons
	Cup (c)	Capacity of a standard measuring cup
	Gallon (gal)	Capacity of a large milk jug

EXAMPLE 1 Choosing the Appropriate Customary Unit

Choose the most appropriate customary unit for each measurement. Justify your answer.

A **the length of a rug**

Feet—the length of a rug is about the length of several sheets of paper.

B **the weight of a magazine**

Ounces—the weight of a magazine is about the weight of several slices of bread.

C **the capacity of an aquarium**

Gallons—the capacity of an aquarium is about the capacity of several large milk jugs.

The following table shows some common equivalent customary units. You can use equivalent measures to convert units of measure.

Length	Weight	Capacity
12 inches (in.) = 1 foot (ft) 3 feet = 1 yard (yd) 5,280 feet = 1 mile (mi) 1,760 yards = 1 mile (mi)	16 ounces (oz) = 1 pound (lb) 2,000 pounds = 1 ton	8 fluid ounces (fl oz) = 1 cup (c) 2 cups = 1 pint (pt) 2 pints = 1 quart (qt) 4 quarts = 1 gallon (gal)

EXAMPLE 2 Converting Customary Units

Convert 19 c to fluid ounces.

Method 1: Use a proportion.

Write a proportion using a ratio of equivalent measures.

$$\frac{\text{fluid ounces}}{\text{cups}} \rightarrow \frac{8}{1} = \frac{x}{19}$$

$$8 \cdot 19 = 1 \cdot x$$

$$152 = x$$

Method 2: Multiply by 1.

Multiply by a ratio equal to 1, and divide out the units.

$$19 \text{ c} = \frac{19 \text{ c}}{1} \times \frac{8 \text{ fl oz}}{1 \text{ c}}$$

$$= \frac{19 \cdot 8 \text{ fl oz}}{1}$$

$$= 152 \text{ fl oz}$$

Nineteen cups is equal to 152 fluid ounces.

EXAMPLE 3 Adding or Subtracting Mixed Units of Measure

A carpenter has a wooden post that is 4 ft long. She cuts 17 in. off the end of the post. What is the length of the remaining post?

First convert 4 ft to inches.

$$\frac{\text{inches}}{\text{feet}} \rightarrow \frac{12}{1} = \frac{x}{4}$$ *Write a proportion using 1 ft = 12 in.*

$$x = 48 \text{ in.}$$

The carpenter cuts off 17 in., so subtract 17 in.

$$4 \text{ ft} - 17 \text{ in.} = 48 \text{ in.} - 17 \text{ in.}$$

$$= 31 \text{ in.}$$

Write the answer in feet and inches.

$$31 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{31}{12} \text{ ft}$$ *Multiply by a ratio equal to 1.*

$$= 2\frac{7}{12} \text{ ft, or 2 ft 7 in.}$$

Think and Discuss

1. Describe an object that you would weigh in ounces.

2. Explain how to convert yards to feet and feet to yards.

4-5 Exercises

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Exercises 1–18, 19, 23, 25, 29, 31, 35, 37

GUIDED PRACTICE

See Example 1

Choose the most appropriate customary unit for each measurement. Justify your answer.

1. the width of a sidewalk
2. the amount of water in a pool
3. the weight of a truck
4. the distance across Lake Erie

See Example 2

Convert each measure.

5. 12 gal to quarts
6. 8 mi to feet
7. 72 oz to pounds
8. 3.5 c to fluid ounces

See Example 3

9. A pitcher contains 4 c of pancake batter. A cook pours out 5 fl oz of the batter to make a pancake. How much batter remains in the pitcher?

INDEPENDENT PRACTICE

See Example 1

Choose the most appropriate customary unit for each measurement. Justify your answer.

10. the weight of a watermelon
11. the wingspan of a sparrow
12. the capacity of a soup bowl
13. the height of an office building

See Example 2

Convert each measure.

14. 28 pt to quarts
15. 15,840 ft to miles
16. 5.4 tons to pounds
17. $6\frac{1}{4}$ ft to inches

See Example 3

18. A sculptor has a 3 lb block of clay. He adds 24 oz of clay to the block in order to make a sculpture. What is the total weight of the clay before he begins sculpting?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP12.

Compare. Write <, >, or =.

19. 6 yd ▢ 12 ft
20. 80 oz ▢ 5 lb
21. 18 in. ▢ 3 ft
22. 5 tons ▢ 12,000 lb
23. 8 gal ▢ 30 qt
24. 6.5 c ▢ 52 fl oz
25. 10,000 ft ▢ 2 mi
26. 20 pt ▢ 40 c
27. 1 gal ▢ 18 c

Helpful Hint
For more on units of time see Skills Bank p. SB8.

28. Grayson has 3 music lessons each week. Each lesson is 45 minutes long. How many total hours will he spend in music lessons in 1 year?
29. **Earth Science** The average depth of the Pacific Ocean is 12,925 feet. How deep is this in miles, rounded to the nearest tenth of a mile?

Order each set of measures from least to greatest.

30. 8 ft; 2 yd; 60 in.

31. 5 qt; 2 gal; 12 pt; 8 c

32. $\frac{1}{2}$ ton; 8,000 oz; 430 lb

33. 2.5 mi; 12,000 ft; 5,000 yd

34. 63 fl oz; 7 c; 1.5 qt

35. 9.5 yd; 32.5 ft; 380 in.

LINK

Agriculture

The winning pumpkin at the 34th annual Pumpkin Weigh-Off in Half Moon Bay, California, weighed 1,524 pounds!

36. Agriculture In one year, the United States produced nearly 895 million pounds of pumpkins. How many ounces were produced by the state with the lowest production shown in the table?

U.S. Pumpkin Production

State	Pumpkins (million pounds)
California	180
Illinois	364
New York	114
Pennsylvania	109

37. Multi-Step A marathon is a race that is 26 miles 385 yards long. What is the length of a marathon in yards?

38. Estimation In 2007, $1 was approximately equal to 1.052 Canadian dollars. About how many Canadian dollars equaled $25?

39. Critical Thinking Explain why it makes sense to divide when you convert a measurement to a larger unit.

40. What's the Error? A student converted 480 ft to inches as follows. What did the student do wrong? What is the correct answer?

$$\frac{1 \text{ ft}}{12 \text{ in.}} = \frac{x}{480 \text{ ft}}$$

41. Write About It Explain how to convert 1.2 tons to ounces.

42. Challenge A dollar bill is approximately 6 in. long. A radio station gives away a prize consisting of a mile-long string of dollar bills. What is the approximate value of the prize?

Test Prep and Spiral Review

43. Multiple Choice Which measure is the same as 32 quarts?

Ⓐ 64 pt Ⓑ 128 gal Ⓒ 16 c Ⓓ 512 fl oz

44. Multiple Choice Judy has 3 yards of ribbon. She cuts off 16 inches of the ribbon to wrap a package. How much ribbon does she have left?

Ⓕ 1 ft 8 in. Ⓖ 4 ft 8 in. Ⓗ 7 ft 8 in. Ⓙ 10 ft 4 in.

45. A store sells a television for $486.50. That price is 3.5 times what the store paid. What was the store's cost? (Lesson 3-5)

Determine whether the ratios are proportional. (Lesson 4-3)

46. $\frac{20}{45}, \frac{8}{18}$ **47.** $\frac{6}{5}, \frac{5}{6}$ **48.** $\frac{11}{44}, \frac{7}{28}$ **49.** $\frac{9}{6}, \frac{27}{20}$

4-6 Metric Measurements

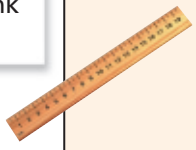
Learn to identify, convert, and compare metric units.

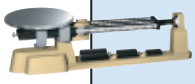
The Micro Flying Robot II is the world's lightest helicopter. Produced in Japan in 2004, the robot is 85 millimeters tall and has a mass of 8.6 grams.

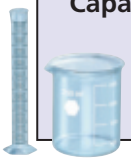
You can use the following benchmarks to help you understand millimeters, grams, and other metric units.

Helpful Hint

For more on metric units, see Skills Bank p. SB7.







	Metric Unit	Benchmark
Length	Millimeter (mm)	Thickness of a dime
	Centimeter (cm)	Width of your little finger
	Meter (m)	Width of a doorway
	Kilometer (km)	Length of 10 football fields
Mass	Milligram (mg)	Mass of a grain of sand
	Gram (g)	Mass of a small paperclip
	Kilogram (kg)	Mass of a textbook
Capacity	Milliliter (mL)	Amount of liquid in an eyedropper
	Liter (L)	Amount of water in a large water bottle
	Kiloliter (kL)	Capacity of 2 large refrigerators

EXAMPLE 1 Choosing the Appropriate Metric Unit

Choose the most appropriate metric unit for each measurement. Justify your answer.

A **The length of a car**

Meters—the length of a car is about the width of several doorways.

B **The mass of a skateboard**

Kilograms—the mass of a skateboard is about the mass of several textbooks.

C **The recommended dose of a cough syrup**

Milliliters—one dose of cough syrup is about the amount of liquid in several eyedroppers.

Reading Math

Prefixes:
Milli- means "thousandth"
Centi- means "hundredth"
Kilo- means "thousand"

The table shows how metric units are based on powers of 10.

$10^3 = 1{,}000$	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$	$\frac{1}{10^1} = 0.1$	$\frac{1}{10^2} = 0.01$	$\frac{1}{10^3} = 0.001$
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
Kilo-	*Hecto-*	*Deca-*	Base unit	*Deci-*	*Centi-*	*Milli-*

To convert metric units, multiply or divide by a power of 10. Multiply to convert to a smaller unit and divide to convert to a larger unit.

EXAMPLE 2 Converting Metric Units

Convert each measure.

A **510 cm to meters**

$510 \text{ cm} = (510 \div 100) \text{ m}$ *100 cm = 1 m, so divide by 100.*

$= 5.1 \text{ m}$ *Since 100 = 10^2, move the decimal point 2 places left: 510.*

B **2.3 L to milliliters**

$2.3 \text{ L} = (2.3 \times 1{,}000) \text{ mL}$ *1 L = 1,000 mL, so multiply by 1,000.*

$= 2{,}300 \text{ mL}$ *Since 1,000 = 10^3, move the decimal point 3 places right: 2.300*

EXAMPLE 3 Using Unit Conversion to Make Comparisons

Mai and Brian are measuring the mass of rocks in their earth science class. Mai's rock has a mass of 480 g. Brian's rock has a mass of 0.05 kg. Whose rock has the greater mass? How much greater is its mass?

$\frac{480}{x} = \frac{1{,}000}{1}$ *Write a proportion.*

$480 = 1{,}000x$ *The cross products are equal.*

$0.48 = x$ *480 g = 0.48 kg*

Since $0.48 \text{ kg} > 0.05 \text{ kg}$, Mai's rock has the greater mass.

$0.48 - 0.05 = 0.43$ *Subtract to find how much greater the mass of Mai's rock is.*

The mass of Mai's rock is 0.43 kg greater than the mass of Brian's rock.

Think and Discuss

1. **Tell** how the metric system relates to the base-10 number system.
2. **Explain** why it makes sense to multiply when you convert to a smaller unit.

4-6 Exercises

Learn It Online
Homework Help Online go.hrw.com, keyword MS10 4-6 Go
Exercises 1–18, 23, 25, 29, 35, 37, 39

GUIDED PRACTICE

See Example 1

Choose the most appropriate metric unit for each measurement. Justify your answer.

1. The mass of a pumpkin
2. The amount of water in a pond
3. The length of an eagle's beak
4. The mass of a penny

See Example 2

Convert each measure.

5. 12 kg to grams
6. 4.3 m to centimeters
7. 0.7 mm to centimeters
8. 3,200 mL to liters

See Example 3

9. On Sunday, Li ran 0.8 km. On Monday, she ran 720 m. On which day did Li run farther? How much farther?

INDEPENDENT PRACTICE

See Example 1

Choose the most appropriate metric unit for each measurement. Justify your answer.

10. The capacity of a teacup
11. The mass of 10 grains of salt
12. The height of a palm tree
13. The distance between your eyes

See Example 2

Convert each measure.

14. 0.067 L to milliliters
15. 1.4 m to kilometers
16. 900 mg to grams
17. 355 cm to millimeters

See Example 3

18. Carmen pours 75 mL of water into a beaker. Nick pours 0.75 L of water into a different beaker. Who has the greater amount of water? How much greater?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP12.

Convert each measure.

19. 1.995 m = ▢ cm
20. 0.00004 kg = ▢ g
21. 2,050 kL = ▢ L
22. 0.002 mL = ▢ L
23. 3.7 mm = ▢ cm
24. 61.8 g = ▢ mg

Compare. Write <, >, or =.

25. 0.1 cm ▢ 1 mm
26. 25 g ▢ 3,000 mg
27. 340 mg ▢ 0.4 g
28. 0.05 kL ▢ 5 L
29. 0.3 mL ▢ 0.005 L
30. 1.3 kg ▢ 1,300 g

31. **Art** The *Mona Lisa* by Leonardo da Vinci is 77 cm tall. *Starry Night* by Vincent Van Gogh is 0.73 m tall. Which is the taller painting? How much taller is it?

Write each set of measures in order from least to greatest.

32. 0.005 kL; 4.1 L; 6,300 mL

33. 1.5 m; 1,200 mm; 130 cm

34. 4,000 mg; 50 kg; 70 g

35. 9.03 g; 0.0008 kg; 1,000 mg

36. Measurement Use a ruler to measure the line segment at right in centimeters. Then give the length of the segment in millimeters and meters.

LINK

Life Science

Bats consume up to 25% of their mass at each feeding.

Life Science **The table gives information about several species of Vesper, or Evening, bats. Use the table for Exercises 37 and 38.**

37. Which bat has the greatest mass?

38. Which bat has a longer wingspread, the Red Bat or the Big Brown Bat? How much longer is its wingspread?

U.S. Vesper Bats		
Name	**Wingspread**	**Mass**
Red Bat	0.3 m	10.9 g
Silver-Haired Bat	28.7 cm	8,500 mg
Big Brown Bat	317 mm	0.01 kg

39. Critical Thinking One milliliter of water has a mass of 1 gram. What is the mass of a liter of water?

40. What's the Error? A student converted 45 grams to milligrams as shown below. Explain the student's error.

$$45 \text{ g} = (45 \div 1{,}000) \text{ mg} = 0.045 \text{ mg}$$

41. Write About It Explain how to decide whether milligrams, grams, or kilograms are the most appropriate unit for measuring the mass of an object.

42. Challenge A decimeter is $\frac{1}{10}$ of a meter. Explain how to convert millimeters to decimeters.

Test Prep and Spiral Review

43. Multiple Choice Which of these is the same as 0.4 grams?

Ⓐ 0.0004 mg Ⓑ 0.004 mg Ⓒ 400 mg Ⓓ 4,000 mg

44. Short Response Which has a greater capacity, a measuring cup that holds 250 milliliters or a measuring cup that holds 0.5 liters? Justify your answer.

Find each value. (Lesson 1-2)

45. 9^2 **46.** 12^0 **47.** 2^7 **48.** 7^3 **49.** 3^4

Use cross products to solve each proportion. (Lesson 4-4)

50. $\frac{80}{x} = \frac{1000}{20}$ **51.** $\frac{a}{5.24} = \frac{28}{2}$ **52.** $\frac{8}{25} = \frac{m}{15}$ **53.** $\frac{2.4}{4} = \frac{8.1}{n}$

4-7 Dimensional Analysis

Learn to use dimensional analysis to make unit conversions.

A **unit conversion factor** is a fraction in which the numerator and denominator represent the same quantity in different units.

For example, $\frac{5{,}280\text{ ft}}{1\text{ mi}}$ is a unit conversion factor. Because 1 mi = 5,280 ft, the conversion factor can be simplified to 1.

Vocabulary

unit conversion factor

$$\frac{5{,}280\text{ ft}}{1\text{ mi}} = \frac{5{,}280\text{ ft}}{5{,}280\text{ ft}} = 1$$

Interactivities Online ▶

You can use a unit conversion factor to change, or convert, measurements from one unit to another. Choosing an appropriate conversion factor is called *dimensional analysis*.

EXAMPLE 1 Using Conversion Factors to Solve Problems

A **As you go deeper underground, the earth's temperature increases. In some places, it may increase by 25 °C per kilometer. Find this rate in degrees per meter.**

Convert the rate 25 °C per *kilometer* to degrees per *meter*.

$\frac{25\text{ °C}}{1\text{ km}} \cdot \frac{1\text{ km}}{1000\text{ m}}$ — *To convert the second quantity in a rate, multiply by a conversion factor with that unit in the first quantity.*

$\frac{25\text{ °C}}{1000\text{ m}}$ — *Divide out like units.* $\frac{^\circ C}{\cancel{km}} \cdot \frac{\cancel{km}}{m} = \frac{^\circ C}{m}$

$\frac{0.025\text{ °C}}{1\text{ m}}$ — *Divide 25 °C by 1000 m.*

The rate is 0.025°C per meter.

B **In the United States in 2003, the average person drank about 22 gallons of milk. Find this rate in quarts per month.**

Convert the rate 22 *gallons* per *year* to *quarts* per *month*.

$\frac{22\text{ gal}}{1\text{ yr}} \cdot \frac{4\text{ qt}}{1\text{ gal}} \cdot \frac{1\text{ yr}}{12\text{ mo}}$ — *To convert, multiply by conversion factors with those units.*

$\frac{22 \cdot 4\text{ qt}}{12\text{ mo}}$ — *Divide out like units.* $\frac{\cancel{gal}}{\cancel{yr}} \cdot \frac{qt}{\cancel{gal}} \cdot \frac{\cancel{yr}}{mo} = \frac{qt}{mo}$

$\frac{88\text{ qt}}{12\text{ mo}}$ — *Multiply.*

$\frac{7.3\text{ qt}}{1\text{ mo}}$ — *Simplify.*

The rate is about 7.3 quarts per month.

Helpful Hint

In Example 1A, "1 km" appears to divide out, leaving "degrees per meter," which are the units asked for. Use this strategy of "dividing out" units when converting rates.

EXAMPLE 2 Converting Between Metric and Customary Units

One inch is 2.54 centimeters. A bookmark has a length of 18 centimeters. What is the length of the bookmark in inches, rounded to the nearest inch?

$\frac{\text{inches}}{\text{centimeters}} \rightarrow \frac{1}{2.54} = \frac{x}{18}$ *Write a proportion using 1 in. = 2.54 cm.*

$1 \cdot 18 = 2.54 \cdot x$ *The cross products are equal.*

$18 = 2.54x$ *Multiply.*

$\frac{18}{2.54} = \frac{2.54x}{2.54}$ *Divide each side by 2.54.*

$7 \approx x$ *Round to the nearest whole number.*

The bookmark is about 7 inches long.

EXAMPLE 3 *Sports Application*

A football player runs from his team's 9-yard line to his team's 44-yard line in 7 seconds. Find the player's average speed in yards per second. Use dimensional analysis to check the reasonableness of your answer.

$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$

$= \frac{35 \text{ yards}}{7 \text{ seconds}}$ *The player runs 44 − 9 = 35 yards in 7 seconds.*

$\frac{35 \text{ yards} \div 7}{7 \text{ seconds} \div 7} = \frac{5 \text{ yards}}{1 \text{ second}}$ *Divide to find yards per second.*

The player's average speed is 5 yards per second.

Convert yd/s to mi/h to see if the answer is reasonable.

$\frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = \frac{3 \text{ mi}}{5280 \text{ yd}} = \frac{1 \text{ mi}}{1760 \text{ yd}}$ *Convert miles to yards.*

$\frac{5 \text{ yd}}{1 \text{ s}} \cdot \frac{1 \text{ mi}}{1760 \text{ yd}} \cdot \frac{3600 \text{ s}}{1 \text{ h}}$ *Set up the conversion factors.*

$= \frac{5 \cancel{\text{yd}}}{1 \cancel{\text{s}}} \cdot \frac{1 \text{ mi}}{1760 \cancel{\text{yd}}} \cdot \frac{3600 \cancel{\text{s}}}{1 \text{ h}}$ *Divide out like units.*

$= \frac{5 \cdot 1 \text{ mi} \cdot 3600}{1 \cdot 1760 \cdot 1 \text{ h}} \approx 10.2 \text{ mi/h}$ *Multiply. Then simplify.*

The player's average speed is approximately 10.2 mi/h, which is a reasonable speed for a football player to run a short distance.

Think and Discuss

1. **Tell** whether you get an equivalent rate when you multiply a rate by a conversion factor. Explain.
2. **Compare** the process of converting feet to inches with the process of converting feet per minute to inches per second.

4-7 Exercises

Learn It Online
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Exercises 1–8, 9, 11, 13, 15

GUIDED PRACTICE

See Example 1

1. The maxmimum speed of the Tupolev Tu-144 airliner is 694 m/s. Find this rate in kilometers per second.

2. Ali's car uses 12 gallons of gas each week. Find this rate in quarts per year.

See Example 2

3. One lap around the Daytona Speedway is 2.5 miles. To the nearest hundredth, how many kilometers is this? (*Hint:* 1 mi ≈ 1.609 km)

See Example 3

4. Martin begins driving to work at 8:15 A.M. He drives 18 miles and arrives at his office at 8:39 A.M. Find Martin's average speed in miles per minute. Use dimensional analysis to check the reasonableness of your answer.

INDEPENDENT PRACTICE

See Example 1

5. Lydia wrote $4\frac{1}{2}$ pages of her science report in one hour. What was her writing rate in pages per minute?

6. An Olympic athlete can run 110 yards in 10 seconds. How fast in miles per hour can the athlete run?

See Example 2

7. One lap around the Talladega Speedway is about 4.3 km. To the nearest tenth, how many miles is one lap around the speedway? (*Hint:* 1 mi ≈ 1.609 km)

See Example 3

8. There are markers every 1000 feet along the side of a road. While driving, Sonya passes marker number 8 at 3:10 P.M. and marker number 20 at 3:14 P.M. Find Sonya's average speed in feet per minute. Use dimensional analysis to check the reasonableness of your answer.

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP12.

Use conversion factors to find each of the following.

9. concert tickets sold in an hour at a rate of 6 tickets sold per minute

10. miles jogged in 1 hour at an average rate of 8.5 feet per second

11. calls made in a 3 day telephone fund-raiser at a rate of 10 calls per hour

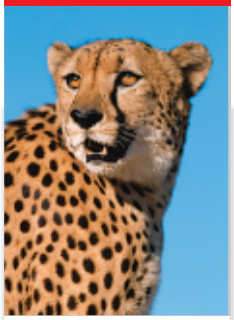
12. **Estimation** In England, a commonly used unit of measure is the *stone*. One stone is equivalent to 14 pounds. Jonathan weighs 95 pounds. About how many stones does he weigh? Round to the nearest tenth of a stone.

13. One pound approximately equals 2.2 kilograms. Water weighs about 62.4 lb per cubic foot. About how much does water weigh in kilograms per cubic foot? Round to the nearest tenth.

14. Ellie added 600 liters of water into a pool in one hour. One liter approximately equals 1.0567 quarts. How many quarts of water per minute did she add? Round to the nearest tenth.

LINK

Life Science



When running at top speed, cheetahs take about 3.5 strides per second. However, a cheetah can maintain this speed for a distance of only 200–300 yards.

15. **Life Science** The Outer Bay exhibit at the Monterey Bay Aquarium holds about 1,000,000 gallons of sea water. How many days would it take to fill the exhibit at a rate of 1 gallon per second?

16. **Money** Fencing costs $3.75 per foot. Bryan wants to enclose his rectangular garden, which measures 6 yards by 4 yards. How much will fencing for the garden cost?

17. **Life Science** A cheetah can run as fast as 70 miles per hour. To the nearest hundredth, what is the cheetah's speed in kilometers per minute?

18. **Transportation** Your car gets 32 miles per gallon of gasoline. Gasoline costs $3 per gallon. How many kilometers can you travel on $30?

19. **Choose a Strategy** Which unit conversion factor should you use to convert 56 square feet to square yards?

a. $\frac{3 \text{ sq ft}}{1 \text{ sq yd}}$ **b.** $\frac{6 \text{ sq ft}}{1 \text{ sq yd}}$ **c.** $\frac{9 \text{ sq ft}}{1 \text{ sq yd}}$ **d.** $\frac{12 \text{ sq ft}}{1 \text{ sq yd}}$

20. **What's the Error?** To convert 5.6 kg to pounds, a student wrote $\frac{5.6 \text{ kg}}{1 \text{ lb}} \cdot \frac{1 \text{ kg}}{2.2 \text{ lb}}$. What error did the student make?

21. **Write About It** Give an example when you would use customary instead of metric measurements, or describe a situation when you would use metric instead of customary measurements.

22. **Challenge** Convert each measure. (*Hint:* 1 oz = 28.35 g)

a. 8 oz = ▢ g **c.** 198.45 g = ▢ oz

b. 538.65 g = ▢ lb **d.** 1.5625 lb = ▢ g

Test Prep and Spiral Review

23. **Multiple Choice** A company rents boats for $9 per hour. How much per minute is this?

Ⓐ $0.15 Ⓑ $0.25 Ⓒ $0.54 Ⓓ $1.05

24. **Multiple Choice** How many square yards are in 27 square feet?

Ⓕ 3 square yards Ⓗ 81 square yards

Ⓖ 9 square yards Ⓙ 243 square yards

25. **Short Response** Show how to convert 1.5 quarts per pound to liters per kilogram. Round each step to the nearest hundredth. (*Hint:* 1L ≈ 1.06 qt, 1 kg ≈ 2.2 lb)

Evaluate each expression for the given value of the variable. (Lesson 1-6)

26. $2x - 3$ for $x = -1$ **27.** $3a + 1$ for $a = 3$ **28.** $3c^2 - 1$ for $c = -3$

Multiply. Write each answer in simplest form. (Lesson 3-9)

29. $12 \cdot \frac{3}{4}$ **30.** $\frac{2}{5} \cdot \left(-\frac{1}{4}\right)$ **31.** $3\frac{2}{3} \cdot \frac{1}{2}$ **32.** $\frac{4}{6} \cdot 10 \cdot 7\frac{1}{2}$

Ready To Go On?

Quiz for Lessons 4-5 Through 4-7

4-5 Customary Measurements

Convert each measure.

1. 7 lb to ounces
2. 15 qt to pints
3. 3 mi to feet
4. 20 fl oz to cups
5. 39 ft to yards
6. 7,000 lb to tons
7. Mara and Andrew are baking cornbread to serve 30 people. They pour 3 cups of milk into the batter and then add 18 more fluid ounces. How much milk did they use?
8. Gabrielle has 3 gal of paint. She uses 9 qt to paint her bedroom. How much paint does she have left?

4-6 Metric Measurements

Convert each measure.

9. 17.3 kg to grams
10. 540 mL to liters
11. 0.46 cm to millimeters
12. 172 L to kiloliters
13. 0.36 km to meters
14. 54.4 mg to grams
15. Cat ran in the 400-meter dash and the 800-meter run. Hilo ran in the 2-kilometer cross-country race. All together, who ran the farthest, Cat or Hilo? How much farther?
16. Luis and Sara collected rainwater over three days. Luis collected 7.6 liters of rainwater, and Sara collected 7,060 milliliters. Who collected more rainwater, Luis or Sara? How much more?

4-7 Dimensional Analysis

17. A yellow jacket can fly 4.5 meters in 9 seconds. What is this rate in meters per minute?
18. The average U.S. citizen throws away about 1,606 lb of trash each year. Find this rate in pounds per month, to the nearest tenth.
19. One gallon is about 3.79 liters. A car has a 55-liter gas tank. What is the capacity of the tank in gallons, rounded to the nearest tenth of a gallon?
20. A 1-pound weight has a mass of about 0.45 kilogram. What is the mass in kilograms of a sculpture that weighs 570 pounds? Round your answer to the nearest tenth of a kilogram.
21. A football player runs from his team's 12-yard line to his team's 36-yard line in 6 seconds. Find the player's average speed in yards per second. Use dimensional analysis to check the reasonableness of your answer.

Focus on Problem Solving

Solve

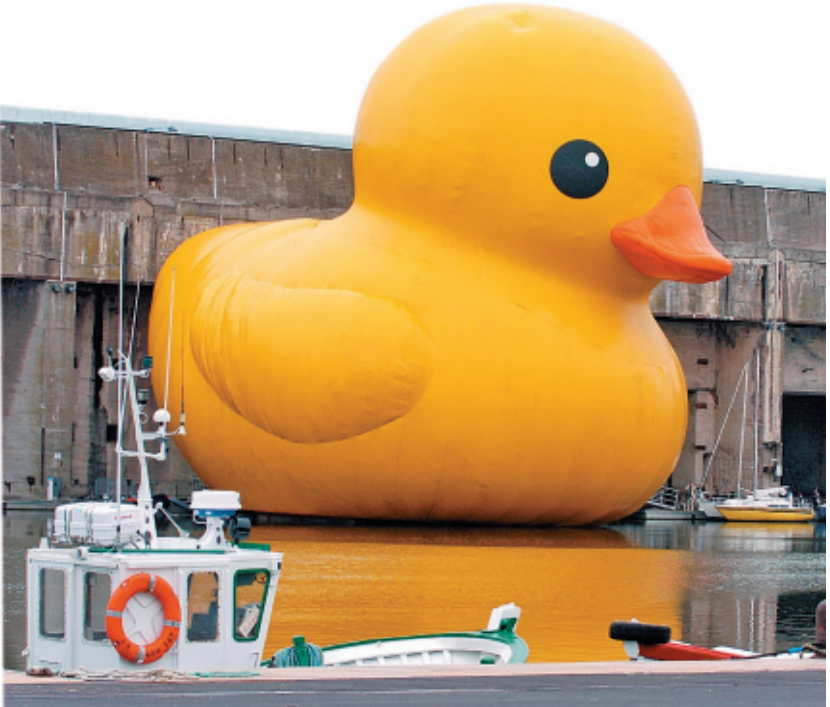
- **Choose an operation: multiplication or division**

When you are converting units, think about whether the number in the answer will be greater than or less than the number given in the question. This will help you decide whether to multiply or divide when changing the units.

Tell whether you would multiply or divide by the conversion factor to solve each problem. Then solve the problem.

1. A pontoon built to look like a duck was part of a 2007 project. The giant yellow duck floated the Loire River in France. Its dimensions were 26 × 20 × 32 meters. Find the dimensions of the duck in feet. Round to the nearest hundredth. (*Hint:* 1 m ≈ 3.28 ft)

2. The length of a rectangle is 8 cm, and its width is 5 cm less than its length. A larger rectangle with dimensions that are proportional to those of the first has a length of 24 cm. What is the width of the larger rectangle in meters?



3. One of the world's largest cheeseburgers was made in Thailand. The cheeseburger weighed 73.6 pounds. It was 23.5 inches in diameter and 13.75 inches in height.

   a. Find the weight of the cheeseburger in kilograms. (*Hint:* 1 lb ≈ 2.2 kg)

   b. Find its dimensions in centimeters. (*Hint:* 1 in. ≈ 2.54 cm)

4. Some of the ingredients for the cheeseburger are listed in the table. Find the missing measures. Round to the nearest hundredth, if necessary.

Cheeseburger Ingredients	Size
Beef	25 kg = ▢ lb
Mustard	$1\frac{1}{2}$ cups = ▢ mL (1 cup ≈ 236.59 mL)
Ketchup	1 cup = ▢ fluid oz

Hands-On LAB 4-8

Make Similar Figures

Use with Lesson 4-8

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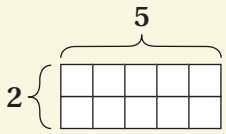
keyword MS10 Lab4 Go

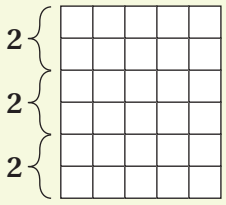
Similar figures are figures that have the same shape but not necessarily the same size. You can make similar rectangles by increasing or decreasing both dimensions of a rectangle while keeping the ratios of the side lengths proportional. Modeling similar rectangles using square tiles can help you solve proportions.

Activity

A rectangle made of square tiles measures 5 tiles long and 2 tiles wide. What is the length of a similar rectangle whose width is 6 tiles?



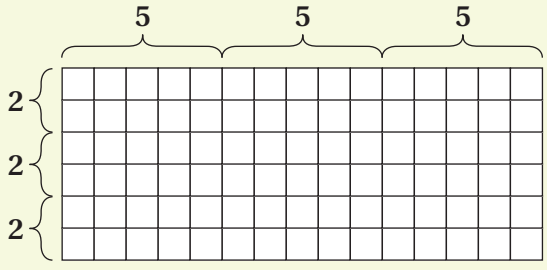
Use tiles to make a 5 × 2 rectangle.



Add tiles to increase the width of the rectangle to 6 tiles.

Notice that there are now 3 sets of 2 tiles along the width of the rectangle because 2 × 3 = 6.

The width of the new rectangle is three times greater than the width of the original rectangle. To keep the ratios of the side measures proportional, the length must also be three times greater than the length of the original rectangle.

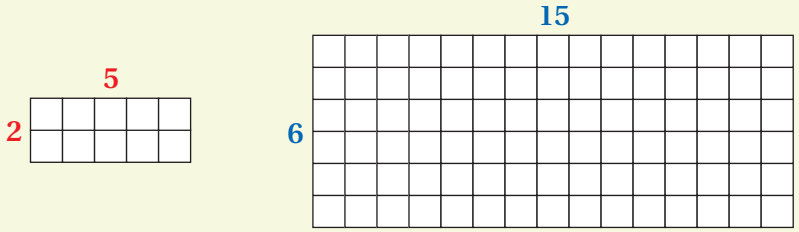


5 × 3 = 15

Add tiles to increase the length of the rectangle to 15 tiles.

The length of the similar rectangle is 15 tiles.

To check your answer, you can use ratios.



$\frac{2}{6} \stackrel{?}{=} \frac{5}{15}$ *Write ratios using the corresponding side lengths.*

$\frac{1}{3} \stackrel{?}{=} \frac{1}{3}$ ✔ *Simplify each ratio.*

1 Use square tiles to model similar figures with the given dimensions. Then find the missing dimension of each similar rectangle.

a. The original rectangle is 4 tiles wide by 3 tiles long. The similar rectangle is 8 tiles wide by x tiles long.

b. The original rectangle is 8 tiles wide by 10 tiles long. The similar rectangle is x tiles wide by 15 tiles long.

c. The original rectangle is 3 tiles wide by 7 tiles long. The similar rectangle is 9 tiles wide by x tiles long.

Think and Discuss

1. Sarah wants to increase the size of her rectangular backyard patio. Why must she change both dimensions of the patio to create a patio similar to the original?

2. In a backyard, a rectangular plot of land that is 5 yd × 8 yd is used to grow tomatoes. The homeowner wants to decrease this plot to 4 yd × 6 yd. Will the new plot be similar to the original? Why or why not?

Try This

1. A rectangle is 3 meters long and 11 meters wide. What is the width of a similar rectangle whose length is 9 meters?

2. A rectangle is 6 feet long and 12 feet wide. What is the length of a similar rectangle whose width is 4 feet?

Use square tiles to model similar rectangles to solve each proportion.

3. $\frac{4}{5} = \frac{8}{x}$

4. $\frac{5}{9} = \frac{h}{18}$

5. $\frac{2}{y} = \frac{6}{18}$

6. $\frac{1}{t} = \frac{4}{16}$

7. $\frac{2}{3} = \frac{8}{m}$

8. $\frac{9}{12} = \frac{p}{4}$

9. $\frac{6}{r} = \frac{9}{15}$

10. $\frac{k}{12} = \frac{7}{6}$

4-8 Similar Figures and Proportions

Learn to use ratios to determine if two figures are similar.

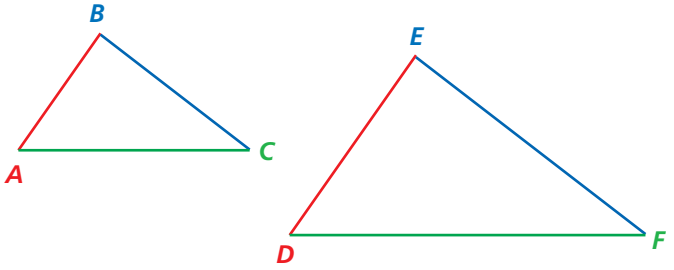
Vocabulary

similar

corresponding sides

corresponding angles

Similar figures are figures that have the same shape but not necessarily the same size. The symbol ~ means "is similar to."



Corresponding angles of two or more similar polygons are in the same relative position. **Corresponding sides** of two or more similar polygons are in the same relative position. When naming similar figures, list the corresponding angles in the same order. For the triangles above, $\triangle ABC \sim \triangle DEF$.

SIMILAR FIGURES

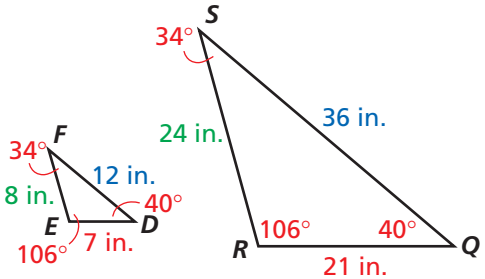
Two figures are similar if

- the measures of their corresponding angles are equal.
- the ratios of the lengths of their corresponding sides are proportional.

EXAMPLE 1 Determining Whether Two Triangles Are Similar

Tell whether the triangles are similar.

The corresponding angles of the figures have equal measures.



$\overline{DE}$ corresponds to $\overline{QR}$.
$\overline{EF}$ corresponds to $\overline{RS}$.
$\overline{DF}$ corresponds to $\overline{QS}$.

$\frac{DE}{QR} \stackrel{?}{=} \frac{EF}{RS} \stackrel{?}{=} \frac{DF}{QS}$ *Write ratios using the corresponding sides.*

$\frac{7}{21} \stackrel{?}{=} \frac{8}{24} \stackrel{?}{=} \frac{12}{36}$ *Substitute the lengths of the sides.*

$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$ *Simplify each ratio.*

Since the measures of the corresponding angles are equal and the ratios of the corresponding sides are equivalent, the triangles are similar.

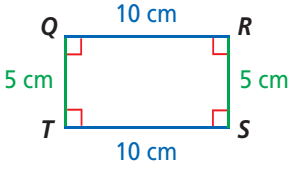
Reading Math

A side of a figure can be named by its endpoints with a bar above, such as $\overline{AB}$. Without the bar, the letters indicate the *length* of the side.

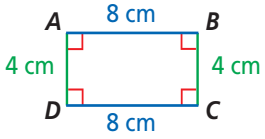
Helpful Hint

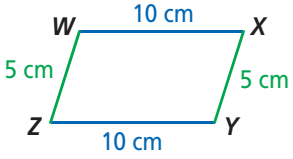
For more on similar triangles, see page SB20 in the Skills Bank.

With triangles, if the corresponding side lengths are all proportional, then the corresponding angles *must* have equal measures. With figures that have four or more sides, if the corresponding side lengths are all proportional, then the corresponding angles *may or may not* have equal angle measures.



ABCD and QRST ***are*** *similar.*

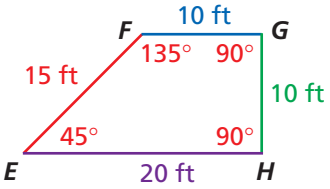


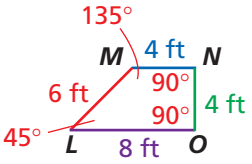


ABCD and WXYZ ***are not*** *similar.*

EXAMPLE 2 Determining Whether Two Four-Sided Figures Are Similar

Tell whether the figures are similar.





The corresponding angles of the figures have equal measures. Write each set of corresponding sides as a ratio.

$\frac{EF}{LM}$ *$\overline{EF}$ corresponds to $\overline{LM}$.* $\frac{FG}{MN}$ *$\overline{FG}$ corresponds to $\overline{MN}$.*

$\frac{GH}{NO}$ *$\overline{GH}$ corresponds to $\overline{NO}$.* $\frac{EH}{LO}$ *$\overline{EH}$ corresponds to $\overline{LO}$.*

Determine whether the ratios of the lengths of the corresponding sides are proportional.

$\frac{EF}{LM} \stackrel{?}{=} \frac{FG}{MN} \stackrel{?}{=} \frac{GH}{NO} \stackrel{?}{=} \frac{EH}{LO}$ *Write ratios using the corresponding sides.*

$\frac{15}{6} \stackrel{?}{=} \frac{10}{4} \stackrel{?}{=} \frac{10}{4} \stackrel{?}{=} \frac{20}{8}$ *Substitute the lengths of the sides.*

$\frac{5}{2} = \frac{5}{2} = \frac{5}{2} = \frac{5}{2}$ *Write the ratios in simplest form.*

Since the measures of the corresponding angles are equal and the ratios of the corresponding sides are equivalent, $EFGH \sim LMNO$.

Think and Discuss

1. **Identify** the corresponding angles of $\triangle JKL$ and $\triangle UTS$.
2. **Explain** whether all rectangles are similar. Give specific examples to justify your answer.

4-8 Exercises

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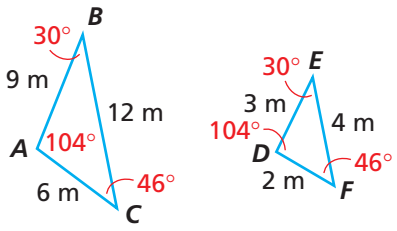
Exercises 1–8, 11, 23

GUIDED PRACTICE

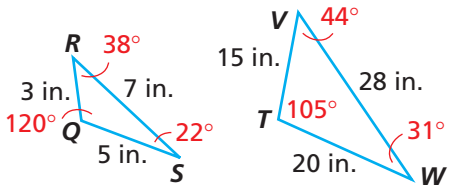
See Example 1

Tell whether the triangles are similar.

1.



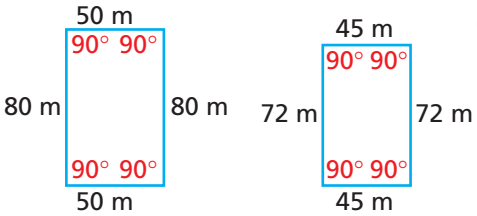
2.



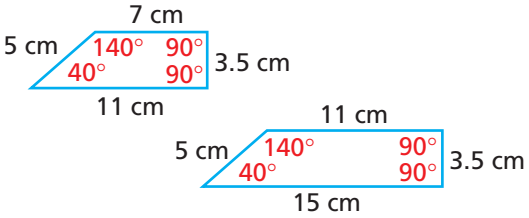
See Example 2

Tell whether the figures are similar.

3.



4.

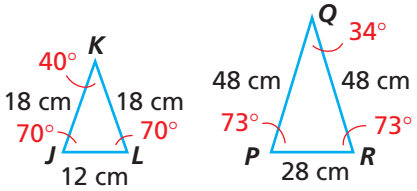


INDEPENDENT PRACTICE

See Example 1

Tell whether the triangles are similar.

5.



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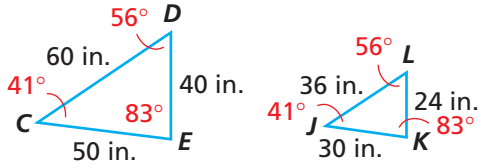
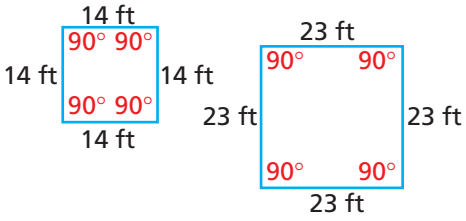


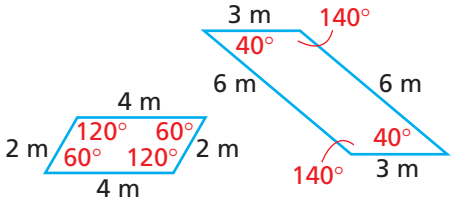
See Example 2

Tell whether the figures are similar.

7.



8.



PRACTICE AND PROBLEM SOLVING

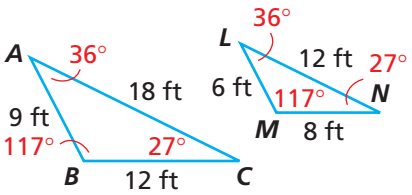
Extra Practice
See page EP13.

9. Tell whether the parallelogram and trapezoid could be similar. Explain your answer.

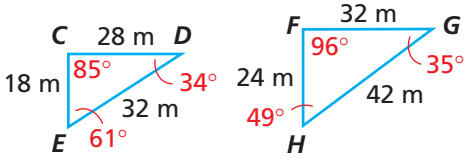
10. Kia wants similar prints in small and large sizes of a favorite photo. The photo lab sells prints in these sizes: 3 in. × 5 in., 4 in. × 6 in., 8 in. × 18 in., 9 in. × 20 in., and 16 in. × 24 in. Which could she order to get similar prints?

Tell whether the triangles are similar.

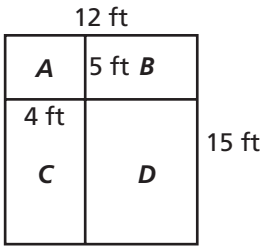
11.



12.



The figure shows a 12 ft by 15 ft rectangle divided into four rectangular parts. Explain whether the rectangles in each pair are similar.



13. rectangle *A* and the original rectangle

14. rectangle *C* and rectangle *B*

15. the original rectangle and rectangle *D*

Critical Thinking **For Exercises 16–19, justify your answers using words or drawings.**

16. Are all squares similar?

17. Are all parallelograms similar?

18. Are all rectangles similar?

19. Are all right triangles similar?

20. **Choose a Strategy** What number gives the same result when multiplied by 6 as it does when 6 is added to it?

21. **Write About It** Tell how to decide whether two figures are similar.

22. **Challenge** Two triangles are similar. The ratio of the lengths of the corresponding sides is $\frac{5}{4}$. The length of one side of the larger triangle is 40 feet. What is the length of the corresponding side of the smaller triangle?

Test Prep and Spiral Review

23. **Multiple Choice** Luis wants to make a deck that is similar to one that is 10 feet long and 8 feet wide. Luis's deck must be 18 feet long. What must its width be?

Ⓐ 20 feet Ⓑ 16 feet Ⓒ 14.4 feet Ⓓ 22.5 feet

24. **Short Response** A real dollar bill measures 2.61 inches by 6.14 inches. A play dollar bill measures 3.61 inches by 7.14 inches. Is the play money similar to the real money? Explain your answer.

Multiply. Write each answer in simplest form. (Lesson 3-9)

25. $-\frac{3}{4} \cdot 14$ **26.** $2\frac{1}{8} \cdot (-5)$ **27.** $\frac{1}{4} \cdot 1\frac{7}{8} \cdot 3\frac{1}{5}$

28. Tell whether 5:3 or 12:7 is a greater ratio. (Lesson 4-1)

4-9 Using Similar Figures

Learn to use similar figures to find unknown measures.

Native Americans of the Northwest carved totem poles out of tree trunks. These poles could stand up to 80 feet tall. Totem poles include carvings of animal figures, such as bears and eagles, which symbolize traits of the family or clan who built them.

Vocabulary

indirect measurement

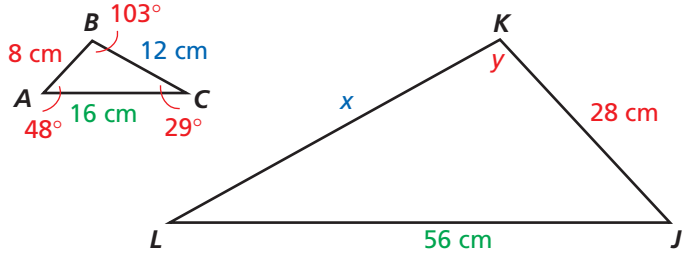
Measuring the heights of tall objects, like some totem poles, cannot be done by using a ruler or yardstick. Instead, you can use *indirect measurement*.

Interactivities Online ▶

Indirect measurement is a method of using proportions to find an unknown length or distance in similar figures.

EXAMPLE 1 Finding Unknown Measures in Similar Figures

$\triangle ABC \sim \triangle JKL$. Find the unknown measures.



Step 1 Find x.

$\frac{AB}{JK} = \frac{BC}{KL}$ *Write a proportion using corresponding sides.*

$\frac{8}{28} = \frac{12}{x}$ *Substitute the lengths of the sides.*

$8 \cdot x = 28 \cdot 12$ *Find the cross products.*

$8x = 336$ *Multiply.*

$\frac{8x}{8} = \frac{336}{8}$ *Divide each side by 8.*

$x = 42$

KL is 42 centimeters.

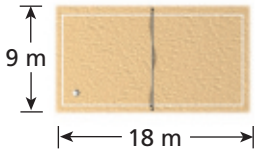
Step 2 Find y.

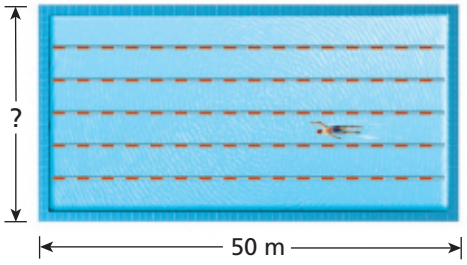
$\angle K$ corresponds to $\angle B$. *Corresponding angles of similar triangles have equal angle measures.*

$y = 103°$

EXAMPLE 2 Measurement Application

A volleyball court is a rectangle that is similar in shape to an Olympic-sized pool. Find the width of the pool.





Let w = the width of the pool.

$\frac{18}{50} = \frac{9}{w}$ *Write a proportion using corresponding side lengths.*

$18 \cdot w = 50 \cdot 9$ *Find the cross products.*

$18w = 450$ *Multiply.*

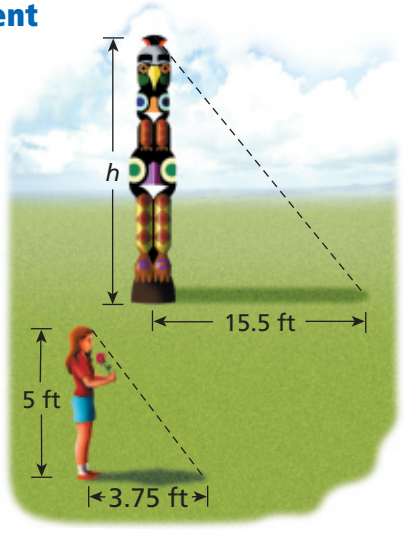
$\frac{18w}{18} = \frac{450}{18}$ *Divide each side by 18.*

$w = 25$

The pool is 25 meters wide.

EXAMPLE 3 Estimating with Indirect Measurement

Estimate the height of the totem pole shown at right.



$\frac{h}{5} = \frac{15.5}{3.75}$ *Write a proportion.*

$\frac{h}{5} \approx \frac{16}{4}$ *Use compatible numbers to estimate.*

$\frac{h}{5} \approx 4$ *Simplify.*

$5 \cdot \frac{h}{5} \approx 5 \cdot 4$ *Multiply each side by 5.*

$h \approx 20$

The totem pole is about 20 feet tall.

Think and Discuss

1. **Write** another proportion that could be used to find the value of x in Example 1.
2. **Name** two objects that it would make sense to measure using indirect measurement.

4-9 Exercises

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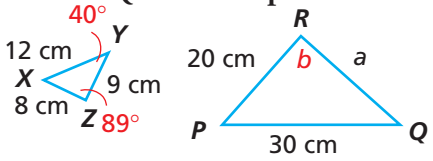
Exercises 1–8, 9, 15

GUIDED PRACTICE

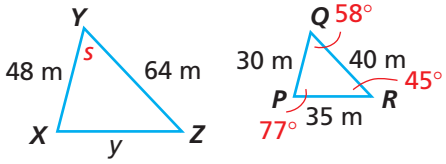
See Example 1

$\triangle XYZ \sim \triangle PQR$ in each pair. Find the unknown measures.

1.

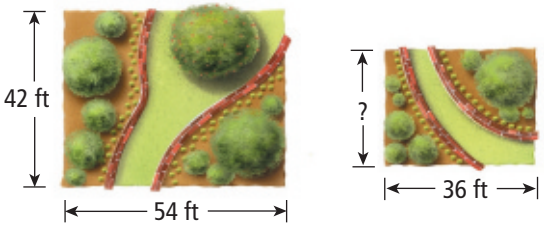


2.



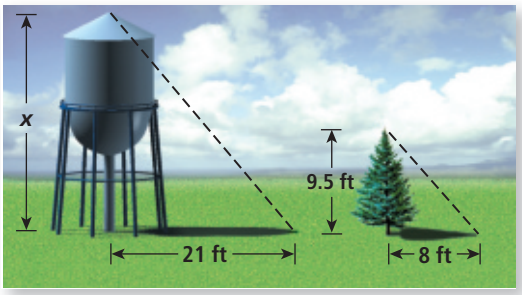
See Example 2

3. The rectangular gardens at right are similar in shape. How wide is the smaller garden?



See Example 3

4. A water tower casts a shadow that is 21 ft long. A tree casts a shadow that is 8 ft long. Estimate the height of the water tower.

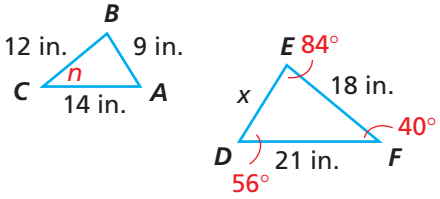


INDEPENDENT PRACTICE

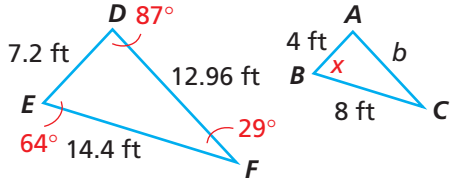
See Example 1

$\triangle ABC \sim \triangle DEF$ in each pair. Find the unknown measures.

5.

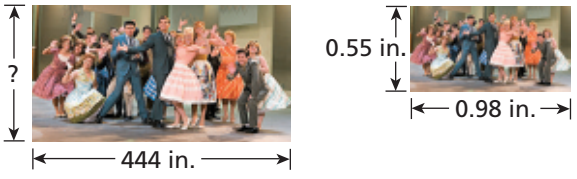


6.



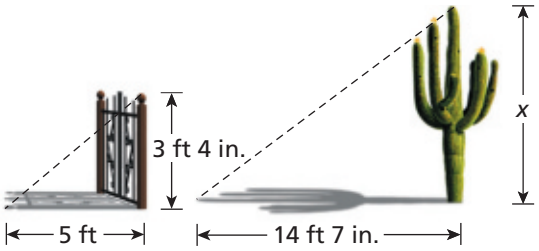
See Example 2

7. The movie still and its projected image at right are similar. What is the height of the projected image to the nearest hundredth of an inch?



See Example 3

8. A cactus casts a shadow that is 14 ft 7 in. long. A gate nearby casts a shadow that is 5 ft long. Estimate the height of the cactus.



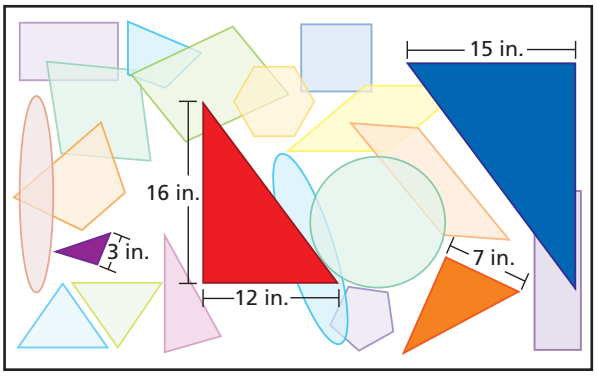
PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP13.

9. A building with a height of 14 m casts a shadow that is 16 m long while a taller building casts a 24 m long shadow. What is the height of the taller building?

10. Two common envelope sizes are $3\frac{1}{2}$ in. × $6\frac{1}{2}$ in. and 4 in. × $9\frac{1}{2}$ in. Are these envelopes similar? Explain.

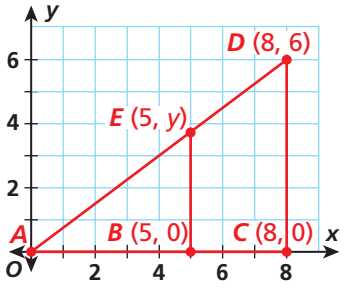
11. Art An art class has painted a mural composed of brightly colored geometric shapes. All of the right triangles in the design are similar to the red right triangle. Find the heights of the three other right triangles in the mural. Round your answers to the nearest tenth.



12. Write a Problem Write a problem that can be solved using indirect measurement.

13. Write About It Assume you know the side lengths of one triangle and the length of one side of a second similar triangle. Explain how to use the properties of similar figures to find the unknown lengths in the second triangle.

14. Challenge $\triangle ABE \sim \triangle ACD$. What is the value of y in the diagram?

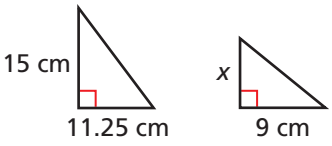


Test Prep and Spiral Review

15. Multiple Choice Find the unknown length in the similar figures.

Ⓐ 10 cm Ⓒ 15 cm

Ⓑ 12 cm Ⓓ 18 cm



16. Gridded Response A building casts a 16-foot shadow. A 6-foot man standing next to the building casts a 2.5-foot shadow. What is the height, in feet, of the building?

Write each phrase as an algebraic expression. (Lesson 1-7)

17. the product of 18 and y **18.** 5 less than a number **19.** 12 divided by z

Choose the most appropriate customary unit for each measurement. Justify your answer. (Lesson 4-5)

20. weight of a cell phone **21.** height of a cat **22.** capacity of a gas tank

4-10 Scale Drawings and Scale Models

Learn to understand ratios and proportions in scale drawings. Learn to use ratios and proportions with scale.

Vocabulary

scale drawing
scale factor
scale model
scale

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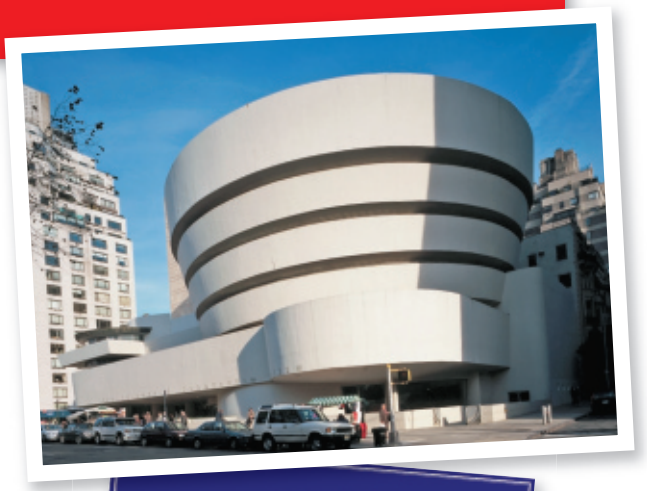

The drawing at right shows a *scale drawing* of the Guggenheim Museum in New York. A **scale drawing** is a proportional two-dimensional drawing of an object. Its dimensions are related to the dimensions of the actual object by a ratio called the **scale factor**. For example, if a drawing of a building has a scale factor of $\frac{1}{87}$, this means that each dimension of the drawing is $\frac{1}{87}$ of the corresponding dimension of the actual building.

A **scale model** is a proportional three-dimensional model of an object. A **scale** is the ratio between two sets of measurements. Scales can use the same units or different units. Both scale drawings and scale models can be smaller or larger than the objects they represent.

EXAMPLE 1 Finding a Scale Factor

Identify the scale factor.

	Race Car	Model
Length (in.)	132	11
Height (in.)	66	5.5

Caution!

A scale factor is always the ratio of the model's dimensions to the actual object's dimensions.

You can use the lengths *or* heights to find the scale factor.

$\frac{\text{model length}}{\text{race car length}} = \frac{11}{132} = \frac{1}{12}$ *Write a ratio. Then simplify.*

$\frac{\text{model height}}{\text{race car height}} = \frac{5.5}{66} = \frac{1}{12}$

The scale factor is $\frac{1}{12}$. This is reasonable because $\frac{1}{10}$ the length of the race car is 13.2 in. The length of the model is 11 in., which is less than 13.2 in., and $\frac{1}{12}$ is less than $\frac{1}{10}$.

EXAMPLE 2 Using Scale Factors to Find Unknown Lengths

A photograph of Rene Magritte's painting *The Schoolmaster* has dimensions 5.4 cm and 4 cm. The scale factor is $\frac{1}{15}$. Find the size of the actual painting.

Think: $\frac{\text{photo}}{\text{painting}} = \frac{1}{15}$

$\frac{5.4}{\ell} = \frac{1}{15}$ *Write a proportion to find the length ℓ.*

$\ell = 5.4 \cdot 15$ *Find the cross products.*

$\ell = 81$ cm *Multiply.*

$\frac{4}{w} = \frac{1}{15}$ *Write a proportion to find the width w.*

$w = 4 \cdot 15$ *Find the cross products.*

$w = 60$ cm *Multiply.*

The painting is 81 cm long and 60 cm wide.

EXAMPLE 3 *Measurement Application*

On a map of Florida, the distance between Hialeah and Tampa is 10.5 cm. The map scale is 3 cm:128 km. What is the actual distance *d* between these two cities?

Think: $\frac{\text{map distance}}{\text{actual distance}} = \frac{3}{128}$

$\frac{3}{128} = \frac{10.5}{d}$ *Write a proportion.*

$3 \cdot d = 128 \cdot 10.5$ *Find the cross products.*

$3d = 1{,}344$

$\frac{3d}{3} = \frac{1{,}344}{3}$ *Divide both sides by 3.*

$d = 448$ km

The distance between the cities is 448 km.

Think and Discuss

1. **Explain** how you can tell whether a model with a scale factor of $\frac{5}{3}$ is larger or smaller than the original object.
2. **Describe** how to find the scale factor if an antenna is 60 feet long and a scale drawing shows the length as 1 foot long.

4-10 Exercises

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Exercises 1–8, 9, 11, 13, 15, 17

GUIDED PRACTICE

See Example 1

Identify the scale factor.

1.

	Grizzly Bear	Model
Height (in.)	84	6

2.

	Moray Eel	Model
Length (ft)	5	$1\frac{1}{2}$

See Example 2

3. In a photograph, a sculpture is 4.2 cm tall and 2.5 cm wide. The scale factor is $\frac{1}{16}$. Find the size of the actual sculpture.

See Example 3

4. Ms. Jackson is driving from South Bend to Indianapolis. She measures a distance of 4.3 cm between the cities on her Indiana road map. The map scale is 1 cm:48 km. What is the actual distance between these two cities?

INDEPENDENT PRACTICE

See Example 1

Identify the scale factor.

5.

	Eagle	Model
Wingspan (in.)	90	6

6.

	Dolphin	Model
Length (cm)	260	13

See Example 2

7. On a scale drawing, a tree is $6\frac{3}{4}$ inches tall. The scale factor is $\frac{1}{20}$. Find the height of the actual tree.

See Example 3

8. **Measurement** On a road map of Virginia, the distance from Alexandria to Roanoke is 7.6 cm. The map scale is 2 cm:80 km. What is the actual distance between these two cities?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP13.

The scale factor of each model is 1:12. Find the missing dimensions.

	Item	Actual Dimensions	Model Dimensions
9.	Lamp	Height: ▢	Height: $1\frac{1}{3}$ in.
10.	Couch	Height: 32 in. Length: 69 in.	Height: ▢ Length: ▢
11.	Table	Height: ▢ Width: ▢ Length: ▢	Height: 6.25 cm Width: 11.75 cm Length: 20 cm

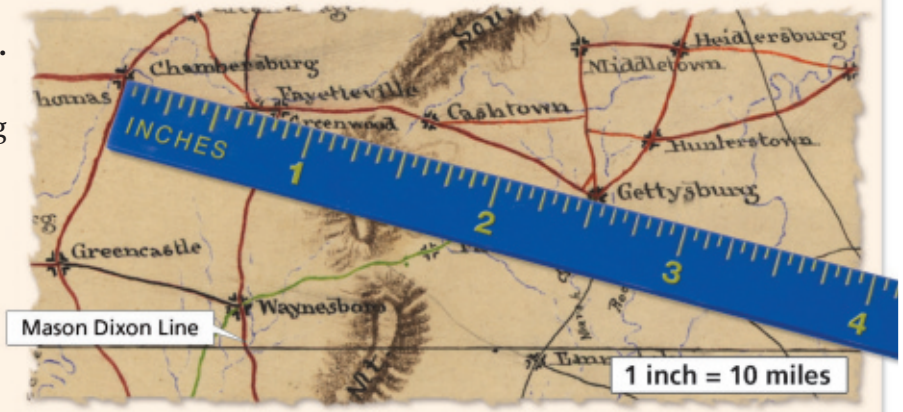
12. An artist transferred a rectangular design 13 cm long and 6 cm wide to a similar canvas 260 cm long and 120 cm wide. What is the scale factor?

13. **Critical Thinking** A countertop is 18 ft long. How long is it on a scale drawing with the scale 1 in:3 yd?

14. **Write About It** A scale for a scale drawing is 10 cm:1 mm. Which will be larger, the actual object or the scale drawing? Explain.

History LINK

Use the map for Exercises 15–16.

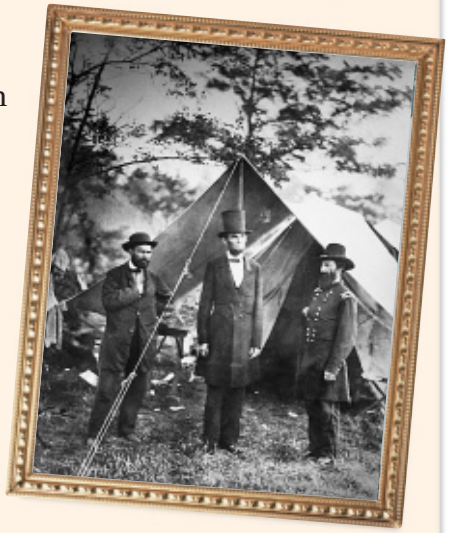


15. In 1863, Confederate troops marched from Chambersburg to Gettysburg in search of badly needed shoes. Use the ruler and the scale of the map to estimate how far the Confederate soldiers, many of whom were barefoot, marched.

16. Before the Civil War, the Mason-Dixon Line was considered the dividing line between the North and the South. Gettysburg is about 8.1 miles north of the Mason-Dixon Line. How far apart in inches are Gettysburg and the Mason-Dixon Line on the map?

17. **Multi-Step** Toby is making a scale model of the battlefield at Fredericksburg. The area he wants to model measures about 11 mi by 7.5 mi. He plans to put the model on a 3.25 ft by 3.25 ft square table. On each side of the model he wants to leave at least 3 in. between the model and the table edges. What is the largest scale he can use?

18. **Challenge** A map of Vicksburg, Mississippi, has a scale of "1 mile to the inch." The map has been reduced so that 5 inches on the original map appears as 1.5 inches on the reduced map. The distance between two points on the reduced map is 1.75 inches. What is the actual distance in miles?



President Abraham Lincoln, Major Allan Pinkerton, and General John A. McCleland, October 1862.

Test Prep and Spiral Review

19. **Multiple Choice** On a scale model with a scale of $\frac{1}{16}$, the height of a shed is 7 inches. What is the approximate height of the actual shed?

Ⓐ 2 feet Ⓑ 9 feet Ⓒ 58 feet Ⓓ 112 feet

20. **Gridded Response** On a map, the scale is 3 centimeters:120 kilometers. The distance between two cities on the map is 6.8 centimeters. What is the distance between the actual cities in kilometers?

Order the numbers from least to greatest. (Lesson 2-11)

21. $\frac{4}{7}$, 0.41, 0.054 **22.** $\frac{1}{4}$, 0.2, -1.2 **23.** 0.7, $\frac{7}{9}$, $\frac{7}{11}$ **24.** 0.3, $-\frac{5}{6}$, 0.32

Divide. Estimate to check whether each answer is reasonable. (Lesson 3-4)

25. $0.32 \div 5$ **26.** $78.57 \div 9$ **27.** $40.5 \div 15$ **28.** $29.68 \div 28$

Hands-On LAB 4-10

Make Scale Drawings and Models

Use with Lesson 4-10

Learn It Online
Lab Resources Online **go.hrw.com**,
keyword MS10 Lab4 Go

Scale drawings and scale models are used in mapmaking, construction, and other trades. You can create scale drawings and models using graph paper. If you measure carefully and convert your measurements correctly, your scale drawings and models will be similar to the actual objects they represent.

Activity 1

Make a scale drawing of a classroom and items with the following dimensions.

Classroom	6 Student Desks	Teacher's Desk	Aquarium
12 ft × 20 ft	2 ft × 3 ft	2 ft × 6 ft	5 ft × 2 ft

1. You can use graph paper for your drawing. When making a scale drawing, you can use any scale you wish. For this activity, use a scale in which 2 squares represent 1 foot. To convert each measurement, multiply the number of feet by 2.
2. This means that the room measures 24 squares (2 • 12 ft) by 40 squares (2 • 20 ft). Convert the other measurements in the table using the same scale.

Classroom	6 Student Desks	Teacher's Desk	Aquarium
24 sq × 40 sq	4 sq × 6 sq	4 sq × 12 sq	10 sq × 4 sq

3. Now sketch the room and items on graph paper. Place the items anywhere in the room you wish.

Think and Discuss

1. Write ratios to compare the widths and lengths of the actual classroom and the drawing. Can you make a proportion with your ratios? Explain.
2. Describe how your drawing would change if you used a scale in which 1 square represents 2 feet.

Try This

1. Measure the dimensions of your classroom as well as some items in the room. Then make a scale drawing. Explain the scale you used.

Activity 2

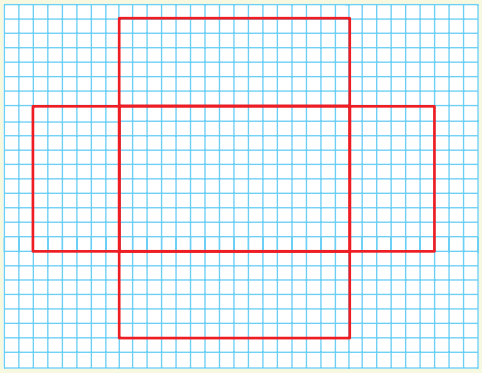
Make a scale model of a school gym whose floor is 20 meters × 32 meters and whose walls are 12 meters tall.

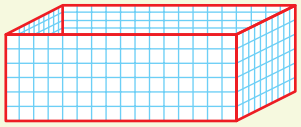
1. You can use graph paper for your model. For this activity, use a scale in which 1 square represents 2 meters. To convert each measurement, divide the number of meters by 2.

2. The two longer sides of the gym floor are 16 squares (32 m ÷ 2). The other two sides are 10 squares (20 m ÷ 2). The walls are 6 squares (12 m ÷ 2) tall.

	Floor Length	Floor Width	Wall Height
Actual	20 m	32 m	12 m
Model	10 squares	16 squares	6 squares

3. Sketch the walls on graph paper as shown. Then cut them out and tape them together to make an open rectangular box to represent the gym.



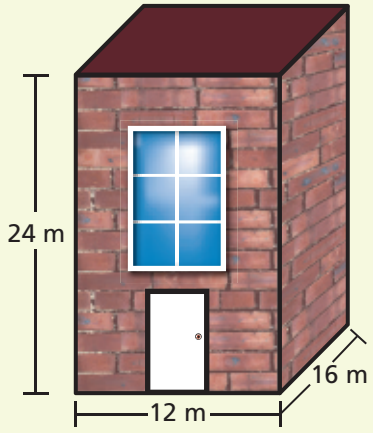


Think and Discuss

1. A different gym has a floor that is 120 feet × 75 feet and a height of 45 feet. A model of the gym has a height of 9 squares. What are the dimensions of the model's floor? What scale was used to create this model?

Try This

1. Make a scale model of the building shown. Explain the scale you used to create your model.

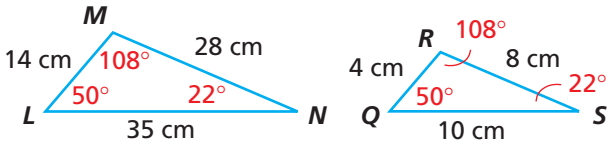


Ready To Go On?

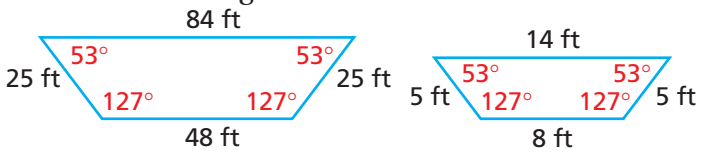
Quiz for Lessons 4-8 Through 4-10

4-8 Similar Figures and Proportions

1. Tell whether the triangles are similar.



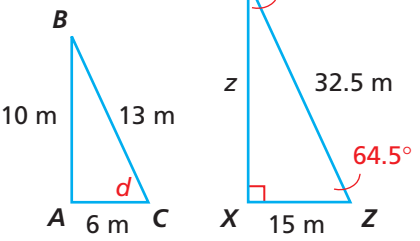
2. Tell whether the figures are similar.



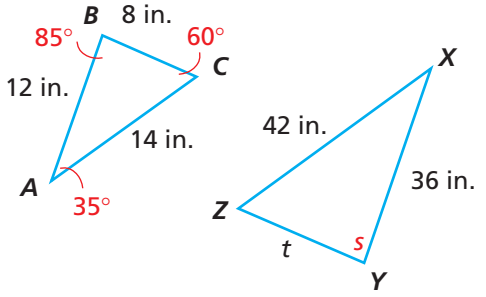
4-9 Using Similar Figures

$\triangle ABC \sim \triangle XYZ$ in each pair. Find the unknown measures.

3.



4.



5. Reynaldo drew a rectangular design that was 6 in. wide and 8 in. long. He used a copy machine to enlarge the rectangular design so that the width was 10 in. What was the length of the enlarged design?

6. Redon is 6 ft 2 in. tall, and his shadow is 4 ft 1 in. long. At the same time, a building casts a shadow that is 19 ft 10 in. long. Estimate the height of the building.

4-10 Scale Drawings and Scale Models

7. An actor is 6 ft tall. On a billboard for a new movie, the actor's picture is enlarged so that his height is 16.8 ft. What is the scale factor?

8. On a scale drawing, a driveway is 6 in. long. The scale factor is $\frac{1}{24}$. Find the length of the actual driveway.

9. A map of Texas has a scale of 1 in:65 mi. The distance from Dallas to San Antonio is 260 mi. What is the distance in inches between these two cities on the map?

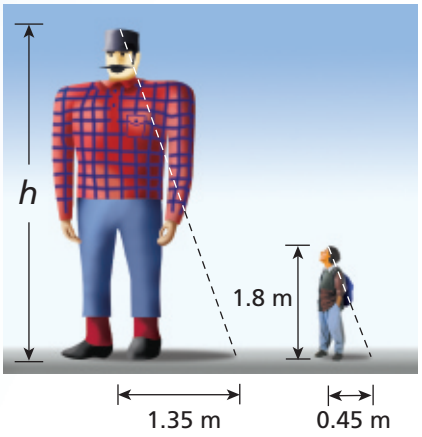
Real-World CONNECTIONS

CHAPTER 4

Paul Bunyan Statues

According to legend, Paul Bunyan was a giant lumberjack whose footsteps created Minnesota's ten thousand lakes. Statues honoring this mythical figure can be found throughout the state. One of the largest, in Brainerd, stands 26 feet tall and can greet you by name!

MINNESOTA



1. A tourist who is 1.8 m tall stands next to the statue of Paul Bunyan in Bemidji, MN. He measures the length of his shadow and the shadow cast by the statue. The measurements are shown in the figure. What is the height of the statue?
2. Show how to use dimensional analysis to convert the height of the statue to feet. Round to the nearest foot. (*Hint*: 1 m = 3.28 ft)
3. The Bemidji statue includes Paul Bunyan's companion, Babe, the Blue Ox. The statue's horns are 14 feet across. The statue was made using the dimensions of an actual ox and a scale of 3:1. What was the length of the horns of the actual ox?
4. The kneeling Paul Bunyan statue in Akeley, MN, is 25 feet tall. The ratio of the statue's height to its width is 17:11. What is the width of the statue to the nearest tenth of a foot?
5. A souvenir of the Akeley statue is made using the scale 2 in:5 ft. What is the height of the souvenir?

Game Time

Water Works

You have three glasses: a 3-ounce glass, a 5-ounce glass, and an 8-ounce glass. The 8-ounce glass is full of water, and the other two glasses are empty. By pouring water from one glass to another, how can you get exactly 6 ounces of water in one of the glasses? The step-by-step solution is described below.

1. Pour the water from the 8 oz glass into the 5 oz glass.
2. Pour the water from the 5 oz glass into the 3 oz glass.
3. Pour the water from the 3 oz glass into the 8 oz glass.

You now have 6 ounces of water in the 8-ounce glass.

Start again, but this time try to get exactly 4 ounces of water in one glass. (*Hint:* Find a way to get 1 ounce of water. Start by pouring water into the 3-ounce glass.)

Next, using 3-ounce, 8-ounce, and 11-ounce glasses, try to get exactly 9 ounces of water in one glass. Start with the 11-ounce glass full of water. (*Hint:* Start by pouring water into the 8-ounce glass.)

Look at the sizes of the glasses in each problem. The volume of the third glass is the sum of the volumes of the first two glasses: $3 + 5 = 8$ and $3 + 8 = 11$. Using any amounts for the two smaller glasses, and starting with the largest glass full, you can get any multiple of the smaller glass's volume. Try it and see.

Concentration

Each card in a deck of cards has a ratio on one side. Place each card face down. Each player or team takes a turn flipping over two cards. If the ratios on the cards are equivalent, the player or team can keep the pair. If not, the next player or team flips two cards. After every card has been turned over, the player or team with the most pairs wins.

A complete copy of the rules and the game pieces are available online.

Learn It Online
Game Time Extra **go.hrw.com**, keyword MS10 Games Go

It's in the Bag!

Materials
- 2 paper plates
- scissors
- markers

PROJECT Paper Plate Proportions

Serve up some proportions on this book made from paper plates.

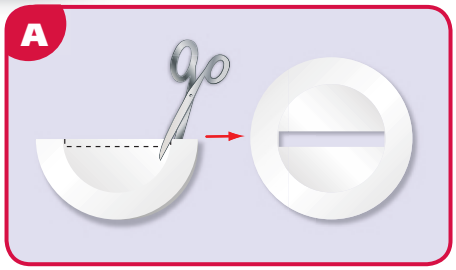
1. Fold one of the paper plates in half. Cut out a narrow rectangle along the folded edge. The rectangle should be as long as the diameter of plate's inner circle. When you open the plate, you will have a narrow window in the center. **Figure A**

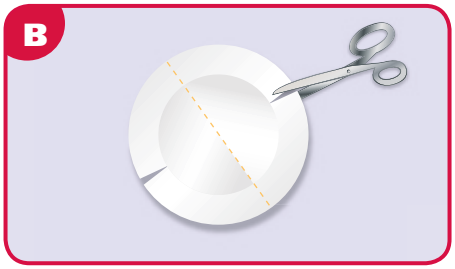
2. Fold the second paper plate in half and then unfold it. Cut slits on both sides of the crease beginning from the edge of the plate to the inner circle. **Figure B**

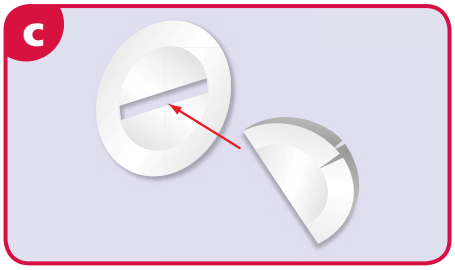
3. Roll up the plate with the slits so that the two slits touch each other. Then slide this plate into the narrow window in the other plate. **Figure C**

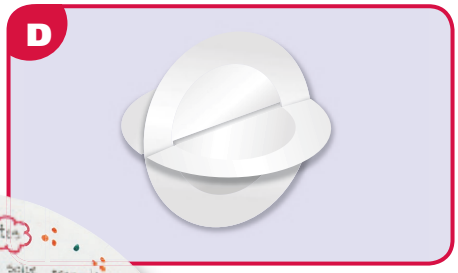
4. When the rolled-up plate is halfway through the window, unroll it so that the slits fit on the sides of the window. **Figure D**

5. Close the book so that all the plates are folded in half.









Taking Note of the Math

Write the number and name of the chapter on the cover of the book. Then review the chapter, using the inside pages to take notes on ratios, rates, proportions, and similar figures.

CHAPTER 4

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. __?__ figures have the same shape but not necessarily the same size.
2. A(n) __?__ is a comparison of two numbers, and a(n) __?__ is a ratio that compares two quantities measured in different units.
3. The ratio used to enlarge or reduce similar figures is a(n) __?__.

EXAMPLES

EXERCISES

4-1 Ratios (pp. 214–217)

■ **Write the ratio of 2 servings of bread to 4 servings of vegetables in all three forms. Write your answers in simplest form.**

$\frac{2}{4} = \frac{1}{2}$ *Write the ratio 2 to 4 in simplest form.*

$\frac{1}{2}$, 1 to 2, 1:2

There are 3 red, 7 blue, and 5 yellow balloons.

4. Write the ratio of blue balloons to total balloons in all three forms. Write your answer in simplest form.
5. Tell whether the ratio of red to blue balloons or the ratio of yellow balloons to total balloons is greater.

4-2 Rates (pp. 218–221)

■ **Find each unit price. Then decide which has the lowest price per ounce.**

$\frac{\$2.70}{5 \text{ oz}}$ or $\frac{\$4.32}{12 \text{ oz}}$

$\frac{\$2.70}{5 \text{ oz}} = \frac{\$0.54}{\text{oz}}$ and $\frac{\$4.32}{12 \text{ oz}} = \frac{\$0.36}{\text{oz}}$

Since $0.36 < 0.54$, $\frac{\$4.32}{12 \text{ oz}}$ has the lowest price per ounce.

Find each unit rate.

6. 540 ft in 90 s
7. 436 mi in 4 hr

Find each unit price. Then decide which is the better buy.

8. $\frac{\$56}{25 \text{ gal}}$ or $\frac{\$32.05}{15 \text{ gal}}$
9. $\frac{\$160}{5 \text{ g}}$ or $\frac{\$315}{9 \text{ g}}$
10. Beatríz earned $197.50 for 25 hours of work. How much money did she earn per hour?

EXAMPLES

EXERCISES

4-3 Identifying and Writing Proportions (pp. 222–225)

■ **Determine if $\frac{5}{12}$ and $\frac{3}{9}$ are proportional.**

$\frac{5}{12}$ — *$\frac{5}{12}$ is already in simplest form.*

$\frac{3}{9} = \frac{1}{3}$ — *Simplify $\frac{3}{9}$.*

$\frac{5}{12} \neq \frac{1}{3}$ — *The ratios are not proportional.*

Determine if the ratios are proportional.

11. $\frac{9}{27}, \frac{6}{20}$ **12.** $\frac{15}{25}, \frac{20}{30}$ **13.** $\frac{21}{14}, \frac{18}{12}$

Find a ratio equivalent to the given ratio. Then use the ratios to write a proportion.

14. $\frac{10}{12}$ **15.** $\frac{45}{50}$ **16.** $\frac{9}{15}$

4-4 Solving Proportions (pp. 226–229)

■ **Use cross products to solve $\frac{p}{8} = \frac{10}{21}$.**

$\frac{p}{8} = \frac{10}{12}$

$p \cdot 12 = 8 \cdot 10$ — *Multiply the cross products.*

$12p = 80$

$\frac{12p}{12} = \frac{80}{12}$ — *Divide each side by 12.*

$p = \frac{20}{3}$, or $6\frac{2}{3}$

Use cross products to solve each proportion.

17. $\frac{4}{6} = \frac{n}{3}$ **18.** $\frac{2}{a} = \frac{5}{15}$

19. $\frac{b}{1.5} = \frac{8}{3}$ **20.** $\frac{16}{11} = \frac{96}{x}$

21. $\frac{2}{y} = \frac{1}{5}$ **22.** $\frac{7}{2} = \frac{70}{w}$

4-5 Customary Measurements (pp. 232–235)

■ **Convert 5 mi to feet.**

$\frac{\text{feet}}{\text{miles}} \rightarrow \frac{5{,}280}{1} = \frac{x}{5}$

$x = 5{,}280 \cdot 5 = 26{,}400$ ft

Convert each measure.

23. 32 fl oz to pt **24.** 1.5 T to lb

25. Manda has 4 yards of fabric. She cuts off 29 inches. What is the length of the remaining fabric?

4-6 Metric Measurements (pp. 236–239)

■ **Convert 63 m to centimeters.**

63 m = (63 × 100) cm — *100 cm = 1 m*

= 6,300 cm

Convert each measure.

26. 18 L to mL **27.** 720 mg to g

28. 5.3 km to m **29.** 0.6 cm to mm

4-7 Dimensional Analysis (pp. 240–243)

■ **Amil can run 12 kilometers in 1 hour. How many meters can he run at this pace in 1 minute?**

km to m: $= \frac{1{,}000 \text{ m}}{1 \text{ km}}$ h to min: $= \frac{1 \text{ h}}{60 \text{ min}}$

$\frac{12 \text{ km}}{1 \text{ h}} \cdot \frac{1{,}000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ h}}{60 \text{ min}} = \frac{12 \cdot 1{,}000 \text{ m}}{60 \text{ min}} = \frac{200 \text{ m}}{1 \text{ min}}$

Use conversion factors to find each rate.

30. 162 lb/yr to lb/mo

31. 1,232 ft/min to mi/h

32. While driving, Abby passed mile marker 130 at 3:10 P.M. and mile marker 170 at 4:00 P.M. Find Abby's average speed in miles per minute.

EXAMPLES

EXERCISES

4-8 Similar Figures and Proportions (pp. 248–251)

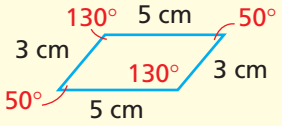
■ **Tell whether the figures are similar.**

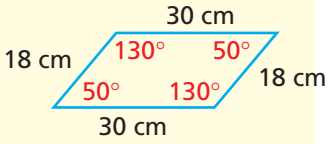
The corresponding angles of the figures have equal measures.

$\frac{5}{30} \stackrel{?}{=} \frac{3}{18} \stackrel{?}{=} \frac{5}{30} \stackrel{?}{=} \frac{3}{18}$

$\frac{1}{6} = \frac{1}{6} = \frac{1}{6} = \frac{1}{6}$

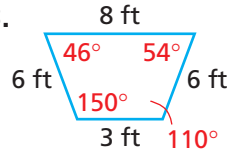
The ratios of the corresponding sides are equivalent. The figures are similar.

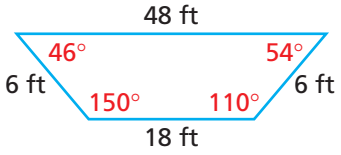




Tell whether the figures are similar.

33.





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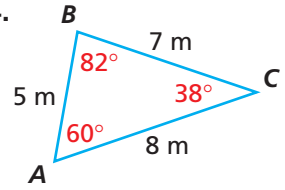
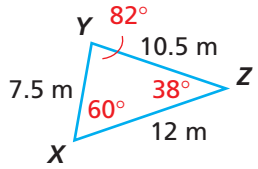




4-9 Using Similar Figures (pp. 252–255)

■ **$\triangle ABC \sim \triangle LMN$. Find the unknown measures.**

$\frac{AB}{LM} = \frac{AC}{LN}$

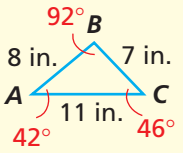
$\frac{8}{t} = \frac{11}{44}$

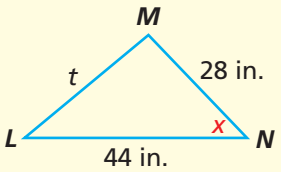
$8 \cdot 44 = t \cdot 11$

$352 = 11t$

$\frac{352}{11} = \frac{11t}{11}$

$32 \text{ in.} = t$



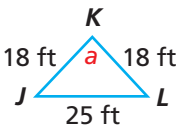


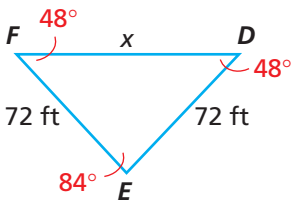
$\angle N$ corresponds to $\angle C$.

$x = 46°$

$\triangle JKL \sim \triangle DEF$. Find the unknown measures.

35.





36. A rectangular photo frame is 24 cm long and 9 cm wide. A frame that is similar in shape is 3 cm wide. Find the length of the frame.

37. A tree casts a $30\frac{1}{2}$ ft shadow at the time of day when a 2 ft stake casts a $7\frac{2}{3}$ ft shadow. Estimate the height of the tree.

4-10 Scale Drawings and Scale Models (pp. 256–259)

■ **A model boat is 4 inches long. The scale factor is $\frac{1}{24}$. How long is the actual boat?**

$\frac{\text{model}}{\text{boat}} = \frac{1}{24}$

$\frac{4}{n} = \frac{1}{24}$ *Write a proportion.*

$4 \cdot 24 = n \cdot 1$ *Find the cross products.*

$96 = n$ *Solve.*

The boat is 96 inches long.

38. The Wright brothers' *Flyer* had a 484-inch wingspan. Carla bought a model of the plane with a scale factor of $\frac{1}{40}$. What is the model's wingspan?

39. The distance from Austin to Houston on a map is 4.3 inches. The map scale is 1 inch:38 miles. What is the actual distance?

Chapter Test

CHAPTER 4

1. Stan found 12 pennies, 15 nickels, 7 dimes, and 5 quarters. Tell whether the ratio of pennies to quarters or the ratio of nickels to dimes is greater.
2. Lenny sold 576 tacos in 48 hours. What was Lenny's average rate of taco sales?
3. A store sells a 5 lb box of detergent for \$5.25 and a 10 lb box of detergent for \$9.75. Which size box has the lowest price per pound?

Find a ratio equivalent to each ratio. Then use the ratios to write a proportion.

4. $\frac{22}{30}$
5. $\frac{7}{9}$
6. $\frac{18}{54}$
7. $\frac{10}{17}$

Use cross products to solve each proportion.

8. $\frac{9}{12} = \frac{m}{6}$
9. $\frac{x}{2} = \frac{18}{6}$
10. $\frac{3}{7} = \frac{21}{t}$
11. $\frac{5}{p} = \frac{10}{2}$

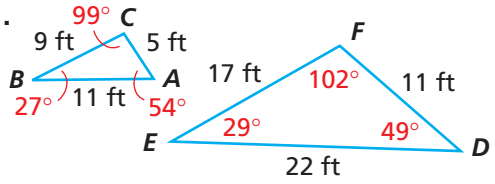
12. A certain salsa is made with 6 parts tomato and 2 parts bell pepper. To correctly make the recipe, how many cups of tomato should be combined with 1.5 cups of bell pepper?

Convert each measure or rate.

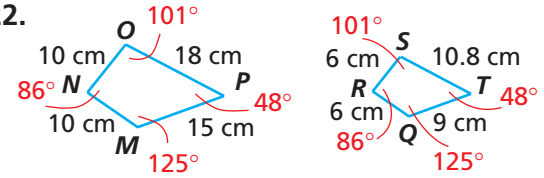
13. 13,200 ft to mi
14. 3.5 lb to oz
15. 6.12 km to m
16. 57 L to kL
17. 828 lb/yr to lb/mo
18. 4.25 L/h to mL/h
19. Some world-class race walkers can walk 9 miles per hour. What is this rate in feet per minute?
20. One pound is about 2.2 kilograms. Jefferson's dog weighs 40 lb. What is the mass of his dog in kilograms?

Tell whether the figures are similar.

21.

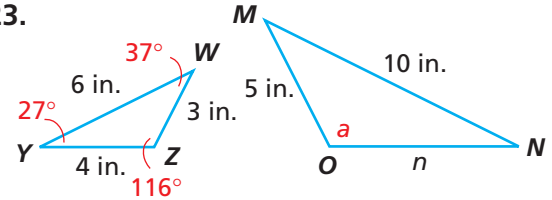


22.



$\triangle WYZ \sim \triangle MNO$ **in each pair. Find the unknown measures.**

23.



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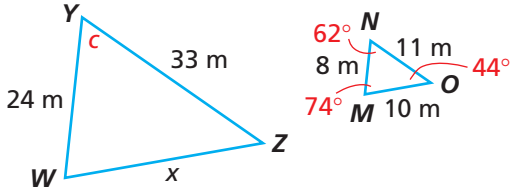

25. A scale model of a building is 8 in. by 12 in. The scale is 1 in:15 ft. What are the dimensions of the actual building?
26. The distance from Portland to Seaside is 75 mi. What is the distance in inches between the two towns on a map whose scale is $1\frac{1}{4}$ in:25 mi?

CHAPTER 4

Standardized Test Prep

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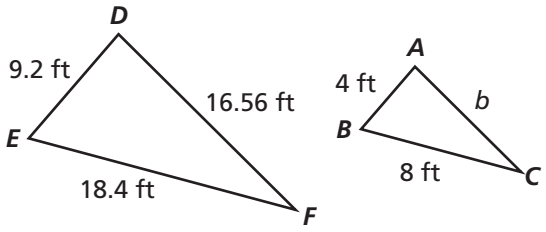
Cumulative Assessment, Chapters 1–4

Multiple Choice

1. What is the unknown length b in similar triangles ABC and DEF?



Ⓐ 7.2 feet
Ⓒ 4 feet
Ⓑ 6 feet
Ⓓ 5.6 feet

2. The total length of the Golden Gate Bridge in San Francisco, California, is 8,981 feet. A car is traveling at a speed of 45 miles per hour. How many minutes will it take the car to cross the bridge?

Ⓕ 0.04 minute
Ⓗ 1.7 minutes
Ⓖ 1.28 minutes
Ⓙ 2.27 minutes

3. For which equation is $x = \frac{2}{5}$ the solution?

Ⓐ $5x - \frac{25}{2} = 0$
Ⓑ $-\frac{1}{5}x + \frac{2}{25} = 0$
Ⓒ $\frac{1}{5}x - 2 = 0$
Ⓓ $-5x + \frac{1}{2} = 0$

4. A hot air balloon descends 38.5 meters in 22 seconds. If the balloon continues to descend at this rate, how long will it take to descend 125 meters?

Ⓕ 25.25 seconds
Ⓗ 71.43 seconds
Ⓖ 86.5 seconds
Ⓙ 218.75 seconds

5. Which value completes the table of equivalent ratios?

Microphones	3	9	15	36
Karaoke Machines	1	3	?	12

Ⓐ 5
Ⓒ 8
Ⓑ 7
Ⓓ 9

6. On a baseball field, the distance from home plate to the pitcher's mound is $60\frac{1}{2}$ feet. The distance from home plate to second base is about $127\frac{7}{24}$ feet. What is the difference between the two distances?

Ⓕ $61\frac{1}{3}$ feet
Ⓗ $66\frac{19}{24}$ feet
Ⓖ $66\frac{5}{6}$ feet
Ⓙ $66\frac{5}{24}$ feet

7. Which word phrase best describes the expression $n - 6$?

Ⓐ 6 more than a number
Ⓑ A number less than 6
Ⓒ 6 minus a number
Ⓓ A number decreased by 6

8. A football weighs about $\frac{3}{20}$ kilogram. A coach has 15 footballs in a large bag. Which is the best description of the total weight of the footballs?

Ⓕ Not quite 3 kilograms
Ⓖ A little more than 2 kilograms
Ⓗ Almost 1 kilogram
Ⓙ Between 1 and 2 kilograms

9. The scale on a map is 1 centimeter: 70 kilometers. The distance between two cities on the map is 8.2 centimeters. Which is the best estimate of the actual distance?

Ⓐ 85 kilometers

Ⓑ 471 kilometers

Ⓒ 117 kilometers

Ⓓ 574 kilometers

10. On a scale drawing, a cell phone tower is 1.25 feet tall. The scale factor is $\frac{1}{150}$. What is the height of the actual cell phone tower?

Ⓕ 37.5 feet

Ⓖ 120 feet

Ⓗ 148 feet

Ⓙ 187.5 feet

HOT TIP! **When a diagram or graph is not provided, quickly sketch one to clarify the information provided in the test item.**

Gridded Response

11. The Liberty Bell, a symbol of freedom in the United States, weighs 2,080 pounds. How many tons does the Liberty Bell weigh?

12. Find the quotient of -51.03 and -8.1.

13. A scale drawing of a rectangular garden has a length of 4 inches and a width of 2.5 inches. The scale is 1 inch:3 feet. What is the perimeter of the actual garden in feet?

14. A florist is preparing bouquets of flowers for an exhibit. The florist has 84 tulips and 56 daisies. Each bouquet will have the same number of tulips and the same number of daisies. How many bouquets can the florist make for this exhibit?

Short Response

S1. Jana began the month with $102.50 in her checking account. During the month, she deposited $8.50 that she earned from baby-sitting, withdrew $9.75 to buy a CD, deposited $5.00 that her aunt gave her, and withdrew $6.50 for a movie ticket. Using compatible numbers, write and evaluate an expression to estimate the balance in Jana's account at the end of the month.

S2. A lamppost casts a shadow that is 18 feet long. At the same time of day, Alyce casts a shadow that is 4.2 feet long. Alyce is 5.3 feet tall. Draw a picture of the situation. Set up and solve a proportion to find the height of the lamppost to the nearest foot. Show your work.

Extended Response

E1. Riley is drawing a map of the state of Virginia. From east to west, the greatest distance across the state is about 430 miles. From north to south, the greatest distance is about 200 miles.

a. Riley is using a map scale of 1 inch: 24 miles. Find the length of the map from east to west and the length from north to south. Round your answers to the nearest tenth.

b. The length between two cities on Riley's map is 9 inches. What is the actual distance between the cities in miles?

c. About how many minutes will it take for an airplane traveling at a speed of 520 miles per hour to fly from east to west across the widest part of Virginia? Show your work.